Presented at the COMSOL Conference 2010 Boston

Computational Methods for Multi-physics Applications with Fluid-structure Interaction

Kumnit Nong and Padmanabhan Seshaiyer
Department of Mathematical Sciences
George Mason University

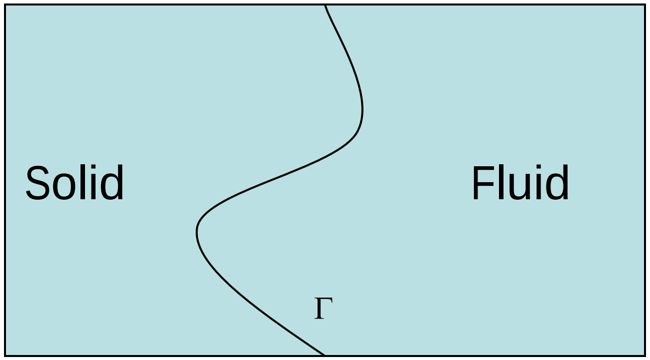
Eugenio Aulisa
Department of Mathematics and Statistics
Texas Tech University

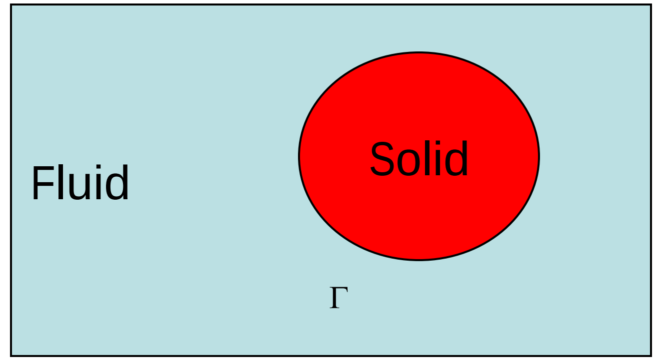
Sonia Garcia
Department of Mathematics
United States Naval Academy

Edward Swim
Department of Mathematics
Sam Houston State University

COMSOL 2010
Boston, USA
(October 8, 2010)

Flow-structure interaction (FSI)





$$Fluid\ :\ \Omega_f \times (0,T)$$

$$\rho_f \frac{\partial \vec{u}}{\partial t} - \nu \Delta \vec{u} + (\vec{u}.\nabla)\vec{u} + \nabla p = \vec{f}$$

$$\nabla.\vec{u} = 0$$

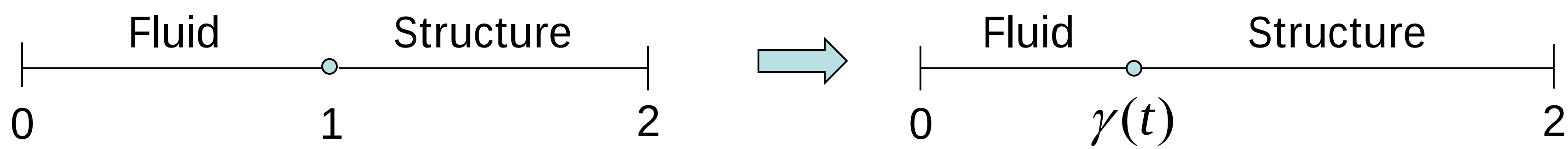
$$Solid\ :\Omega_s \times (0,T)$$

$$\rho_s \frac{\partial^2 w}{\partial t^2} - \nabla.\tilde{\sigma} = \vec{b}$$

$$\tilde{\sigma} = \lambda\, tr(\tilde{\varepsilon}) + 2\mu\tilde{\varepsilon}$$

$$\tilde{\varepsilon} = 0.5[\nabla w + (\nabla w)^T]$$

Fluid-Structure Interaction (1D)



$$\rho_f \frac{\partial u}{\partial t} - \mu_f \frac{\partial^2 u}{\partial x^2} + \frac{3}{2}\rho_f u \frac{\partial u}{\partial x} = f(t,x) \qquad x \in (0,\, 1)$$

$$\rho_s \frac{\partial^2 d}{\partial t^2} - \mu_s \frac{\partial^2 d}{\partial x^2} = g(t,x) \qquad x \in (1,2)$$

$$\gamma(t) = 1 + d(t,1) \qquad t \geq 0$$

$$u(t,\gamma(t)) = \frac{\partial d}{\partial t}(t,1)$$

$$\mu_f \frac{\partial u}{\partial x}(t,\gamma(t)) = \mu_s \frac{\partial d}{\partial x}(t,1)$$

ALE Formulation

$$w(t,x) = \frac{x}{\gamma(t)}\,\dot{\gamma}(t) \qquad\qquad x \in (0,\gamma(t))$$

For all $\xi \in (0,\gamma(s))$:

$$\frac{dx_s}{dt}(t,\xi) = w(t, x_s(t,\xi))$$

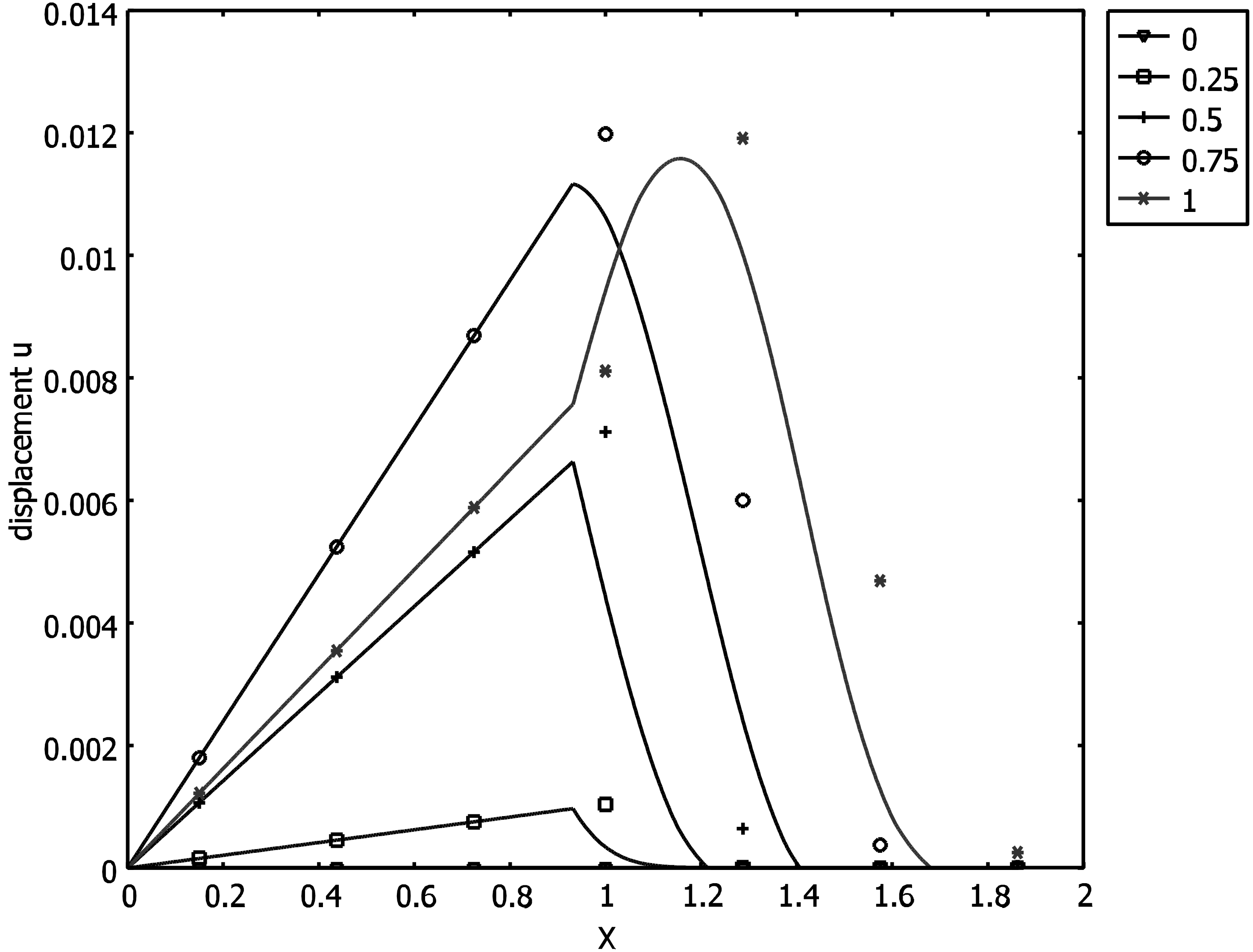
$$x_s(s,\xi) = \xi$$

⇓

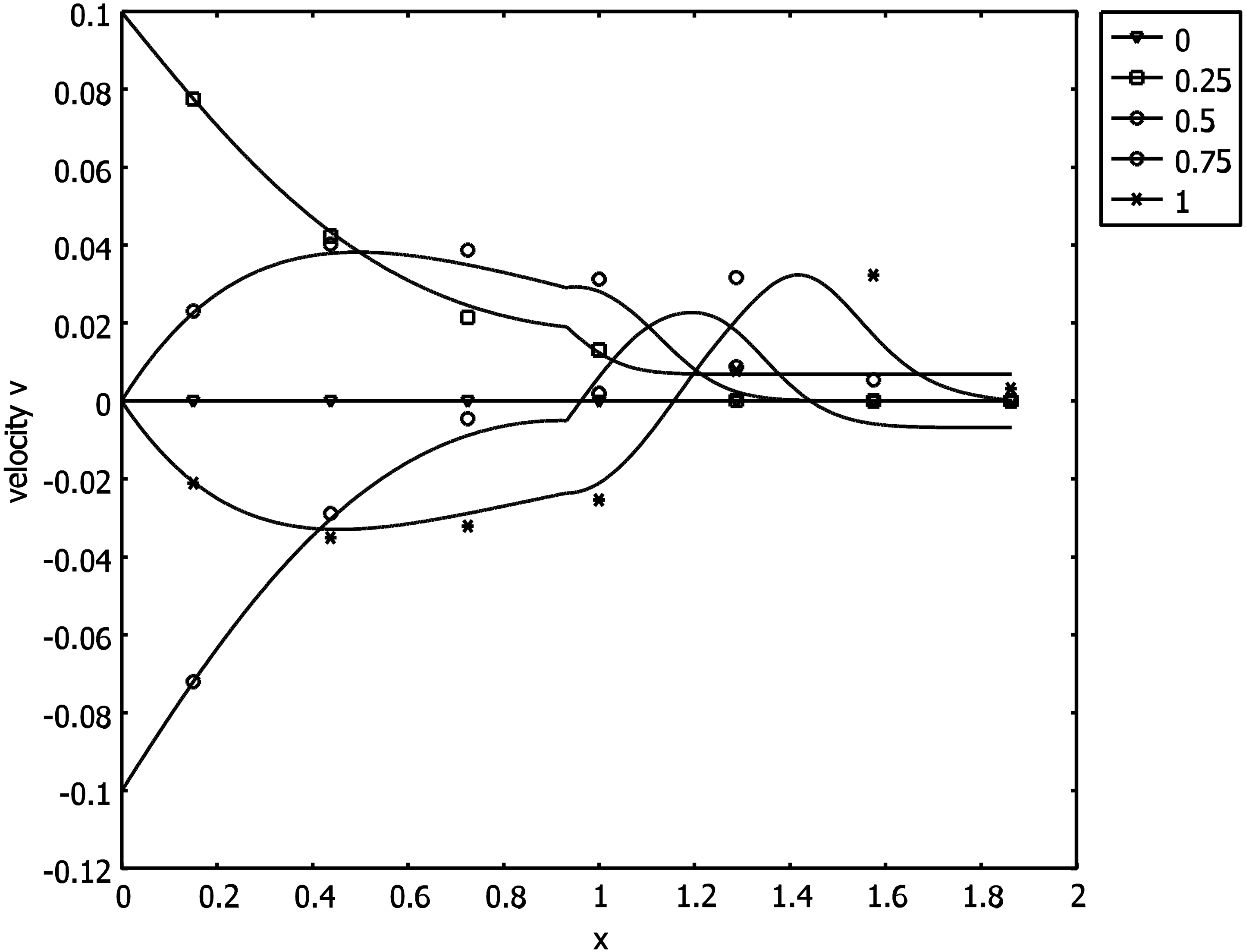
$$x_s(t,\xi) = \xi\,\frac{\gamma(t)}{\gamma(s)}$$

⇓

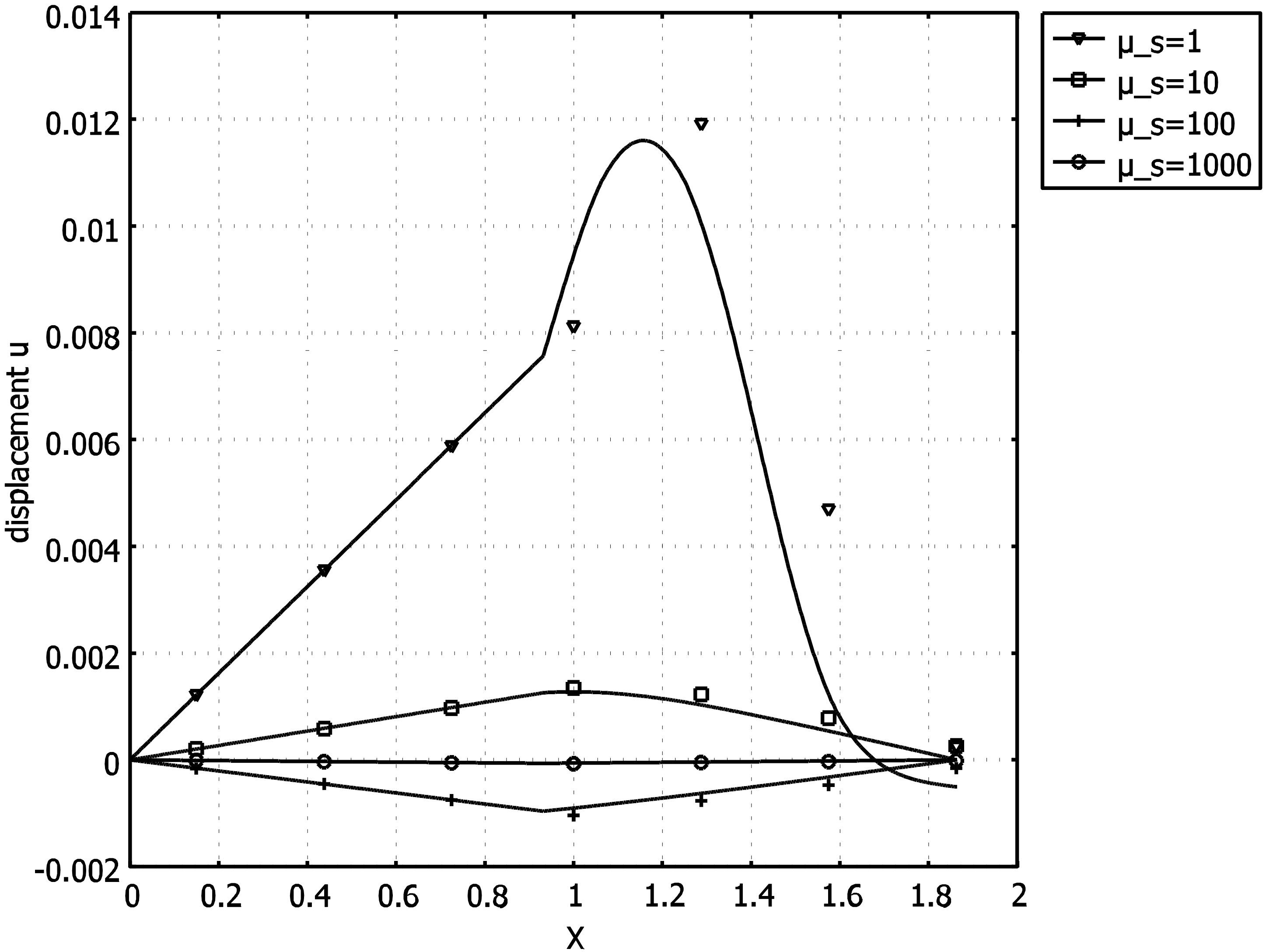
$$\frac{\partial v}{\partial t}(t,\xi) - \mu_f\,\frac{\partial^2 u}{\partial x^2}(t, x_s(t,\xi)) + [1.5u - w]\frac{\partial u}{\partial x}(t, x_s(t,\xi)) = f(t, x_s(t,\xi))$$



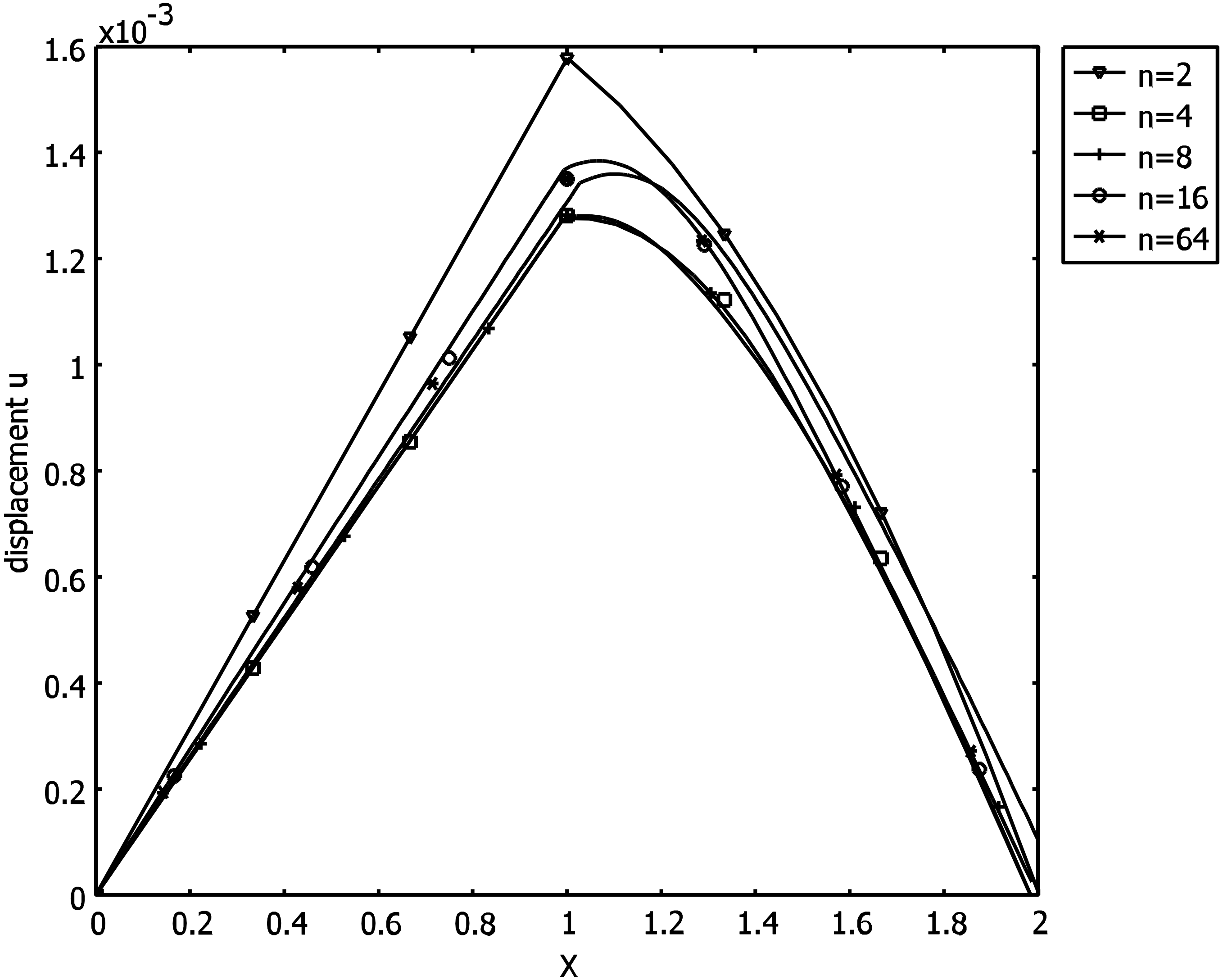
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
displacement u
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
X
0
0.25
0.5
0.75
1



0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
-0.12
velocity v
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
x
0
0.25
0.5
0.75
1



0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
-0.002
displacement u
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
X
μ_s=1
μ_s=10
μ_s=100
μ_s=1000


x10^-3
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
displacement u
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
X
n=2
n=4
n=8
n=16
n=64

Coupled FSI with Control

$$M = \int_0^T \int_{-1}^{\gamma(t)} \left(\frac{1}{2}\left(u - \hat{u}\right)^2 + \frac{1}{2}\alpha_f \; f_f^2 \right)$$

$$+ \int_0^T \int_{-1}^{0} \left(\frac{1}{2}\left(d - \hat{d}\right)^2 + \frac{1}{2}\alpha_s \; f_s^2 \right)$$

$$+ \int_0^T \int_{-1}^{\gamma(t)} \left(l(\rho_f u_t - \mu_f u_{xx} + 1.5 u u_x - f_f) \right)$$

$$+ \int_0^T \int_{-1}^{0} \left(g(\rho_s d_{tt} - \mu_s d_{xx} - f_s) \right)$$

Auxiliary system of PDEs

- In the fluid domain:

$$\rho_f u_t - \mu_f u_{xx} + 1.5 u u_x - \frac{l}{\alpha_f} = 0$$

$$-\rho_f u_t - \mu_f l_{xx} - 1.5 u l_x + u - \hat{u} = 0$$

- In the solid domain:

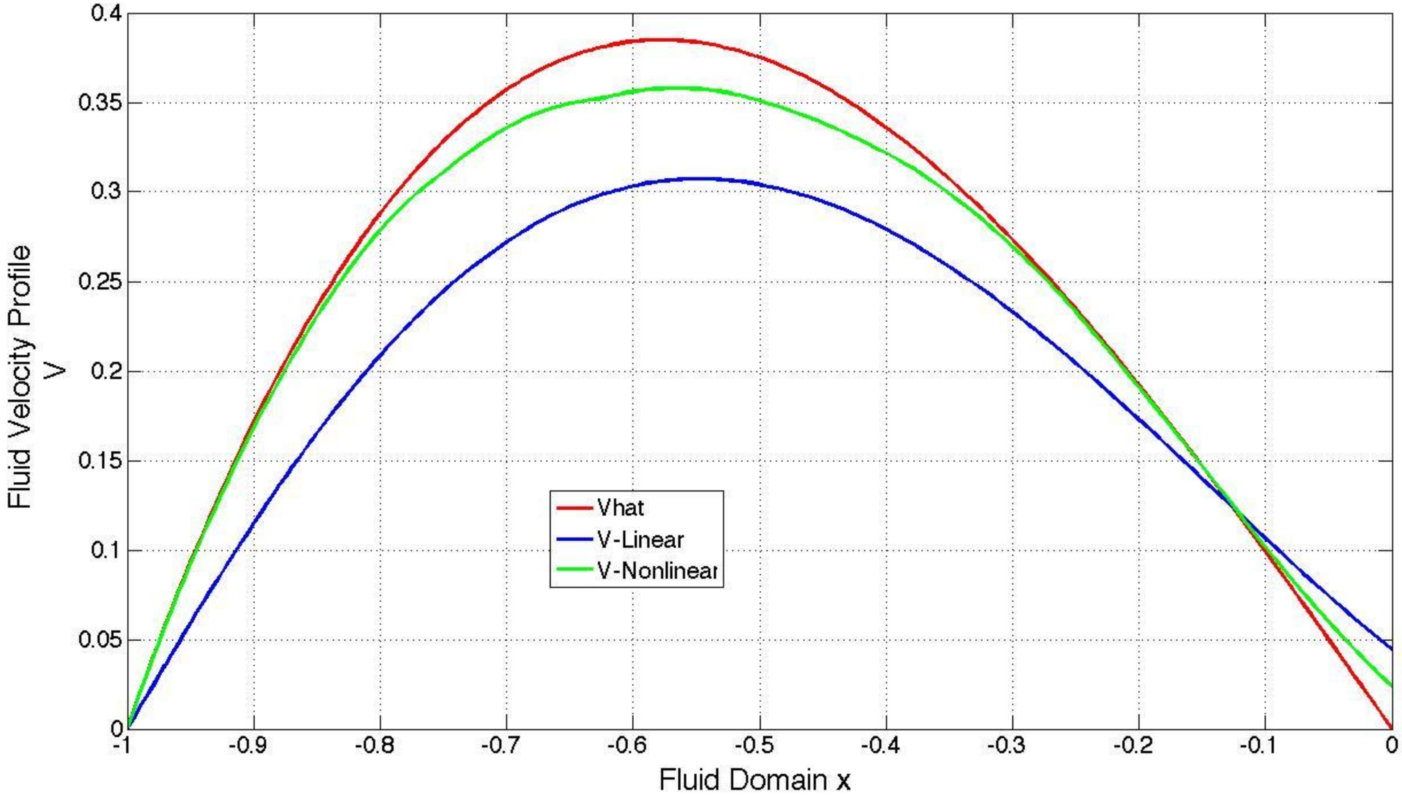
$$\rho_s u_t - \mu_s d_{xx} - \frac{g}{\alpha_s} = 0$$

$$\rho_s g_{tt} - \mu_s g_{xx} + d - \hat{d} = 0$$

Fluid-velocity Profile

$$\hat{d} = 0.5x(x^2 - 1)t^2$$

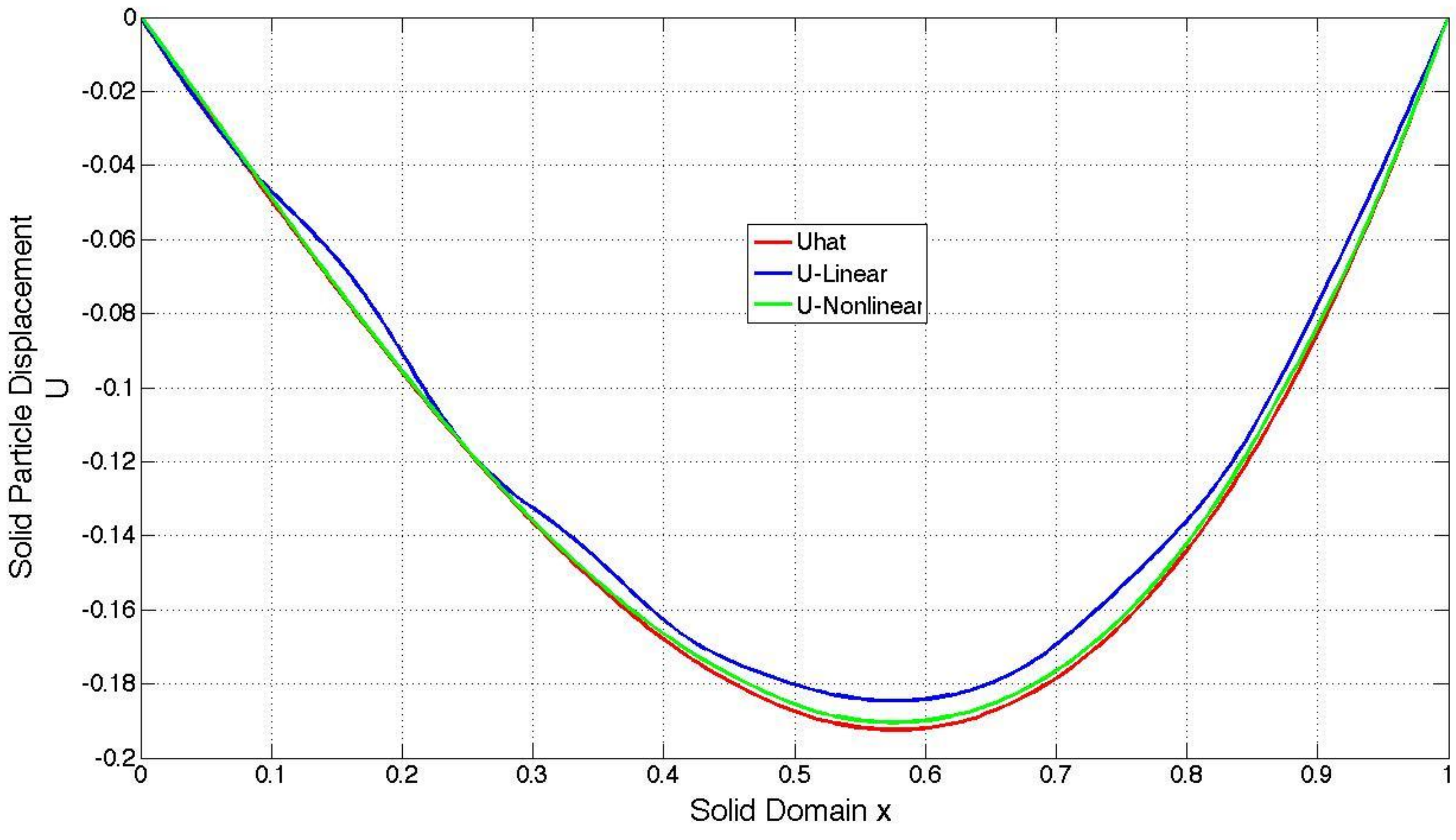
$$\hat{u} = x(x^2 - 1)t$$



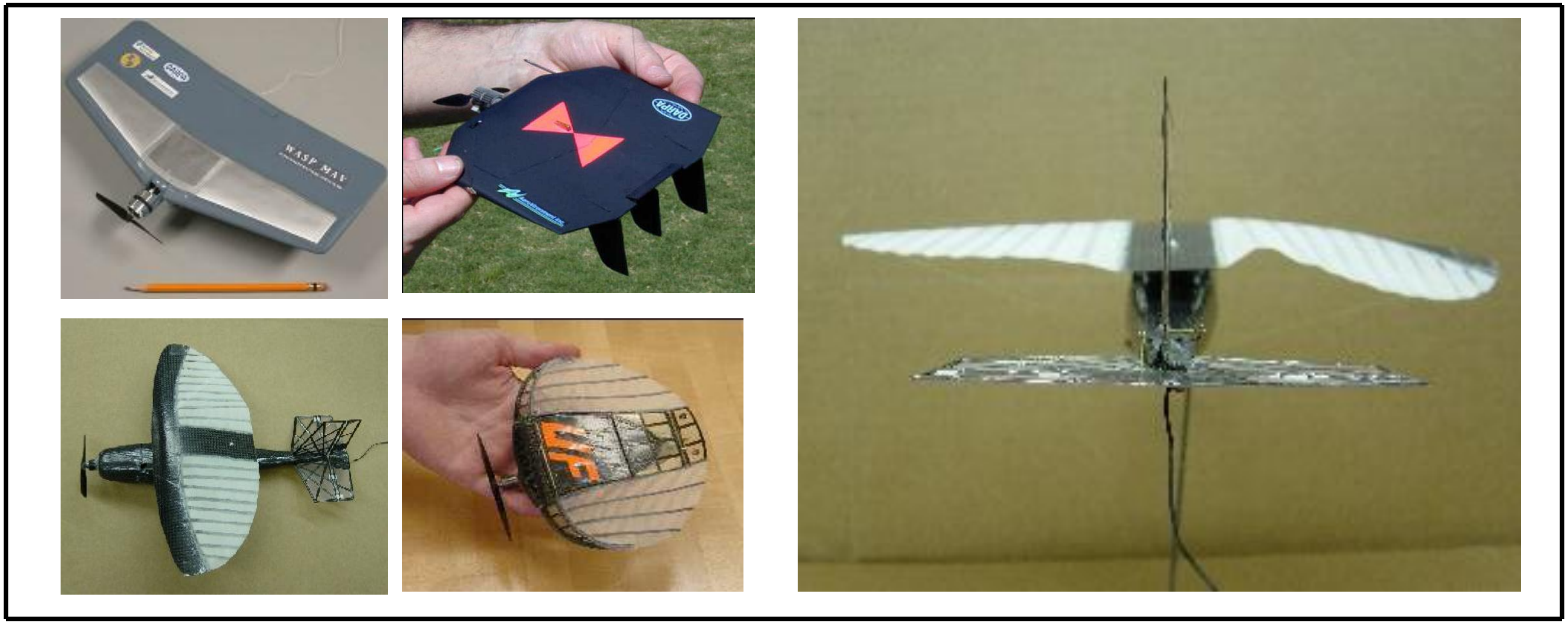
Displacement Profile

$$\hat{d} = 0.5x(x^2 - 1)t^2$$

$$\hat{u} = x(x^2 - 1)t$$



MAV: Membrane Wing Deflection

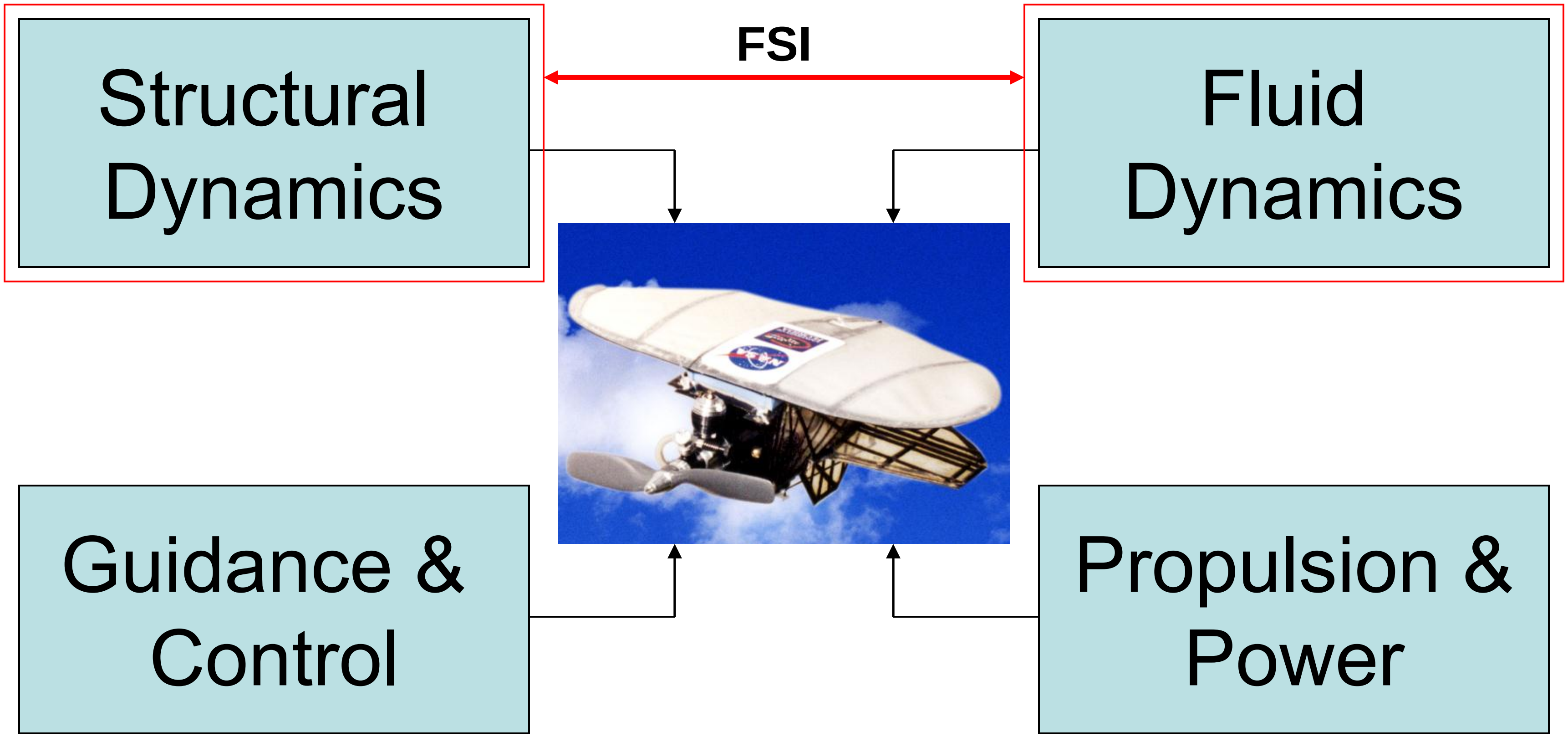


Computational modeling of highly flexible membrane wings for MAVs
Ferguson, L., Aulisa, E., Seshaiyer P. and Gordnier R., (AIAA 2006-1661)
Computational modeling of coupled membrane-beam flexible wings for MAVs
L. Ferguson, P. Seshaiyer, R. Gordnier, P. Attar (AIAA 2007-1787)
Nonlinear Models for Biologically- Inspired Elastic Membrane Wings
E. Swim and P. Seshaiyer (AIAA-2008-2008)

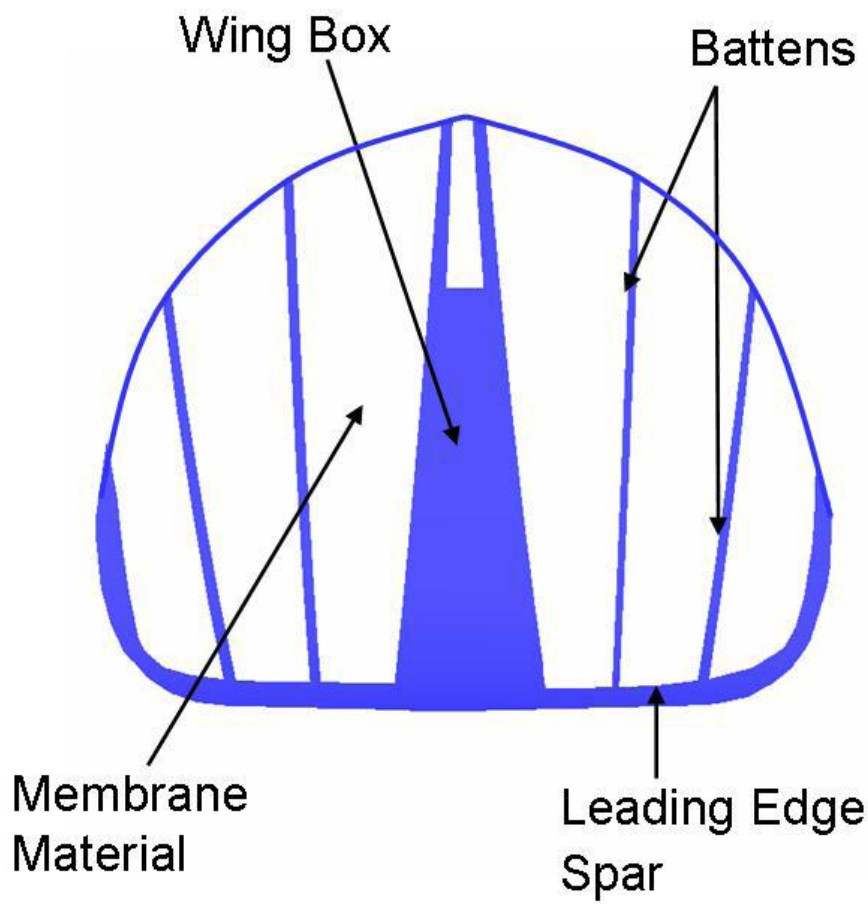
Experimental Challenges in MAV Design

- Small size
- High surface-to-volume ratio
- Constrained weight and volume limitations
- Low Reynolds number regime
- Low aspect ratio fixed to rotary to flapping wings
- Longer flight time
- Better range-payload performance

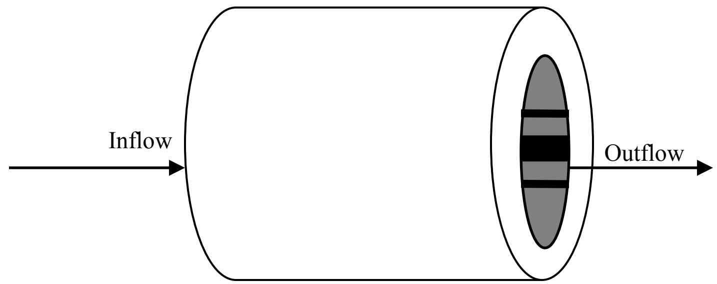
Computational Challenges in MAV Design



Model of a flexible MAV wing



Computational Model for MAV



$$\Delta\phi = 0 \quad in \quad \Omega_3$$
$$\nabla\phi \cdot \vec{n} = 0 \quad on \quad \Gamma_f^N$$

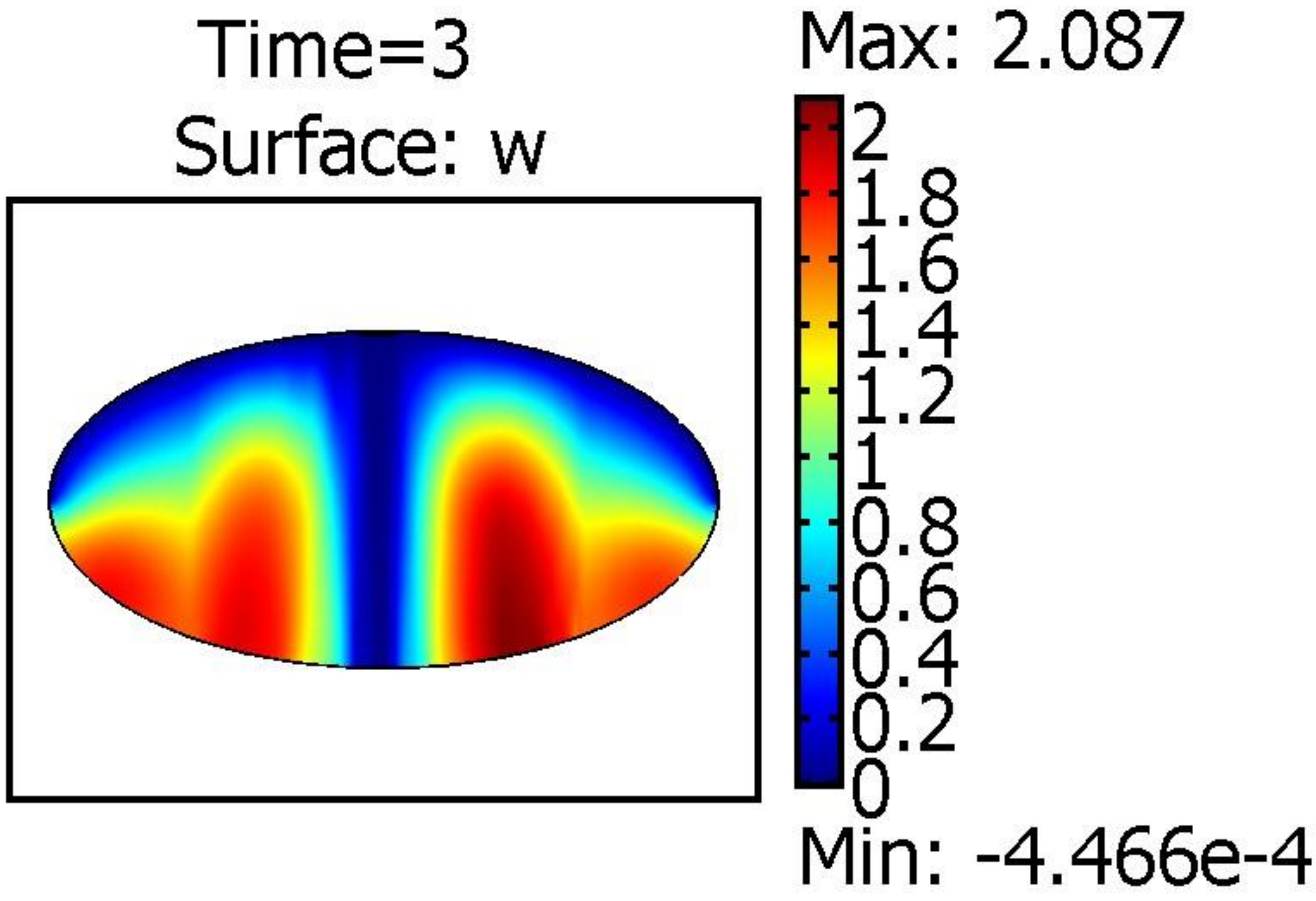
$$\rho_0 \, w_{tt} - E_0 \, \Delta w = f \qquad in \quad \Omega_1$$
$$(\rho_0 + \rho_1) w_{tt} - E_0 \, \Delta w + E_1 \, v_{yy} = -\rho_f \, \phi_t \quad in \quad \Omega_2$$
$$v = w_{yy} + \varepsilon \, \Delta v \qquad in \quad \Omega_1 \cup \Omega_2$$

$$\nabla\phi \cdot \vec{n} = -a \, \phi_t \quad on \quad \Gamma_f^O$$

$$\nabla\phi \cdot \vec{n} = -0.1 + 0.025 \sin(2\pi t) \quad on \quad \Gamma_f^I$$

$$\nabla\phi \cdot \vec{n} = w_t \quad on \quad \Omega_1$$

Membrane Wing Deflection



Membrane Wing Deflection