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Modeling light diffraction using COMSOL

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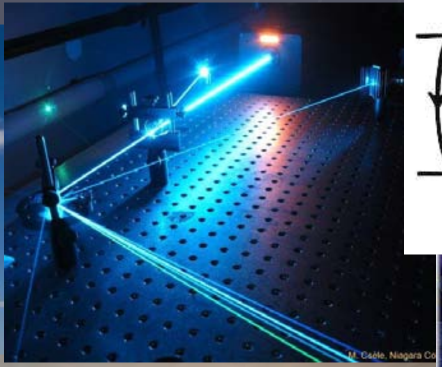
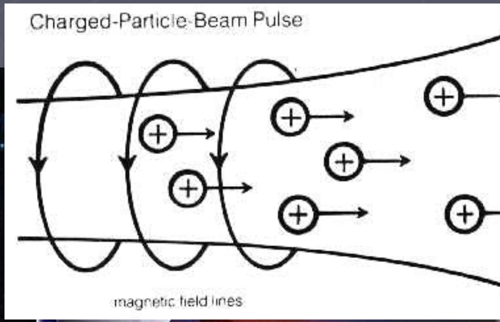
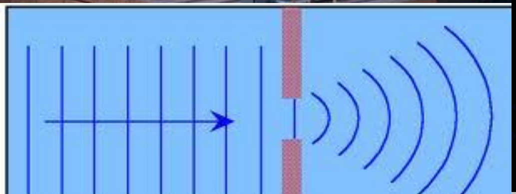
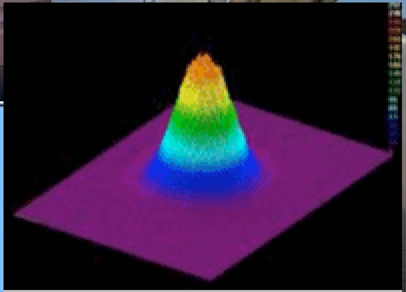
Directed Energy

- Directed Energy: Controlling or manipulating electromagnetic radiation, microwaves, or particle beams in a fashion to achieve a certain end state.
- Studied since the 1960's
- Navy has much interest in Directed Energy
- Speed of light, unlimited energy source
- Major Difficulty: Atmospheric Propagation

Office of Naval Research

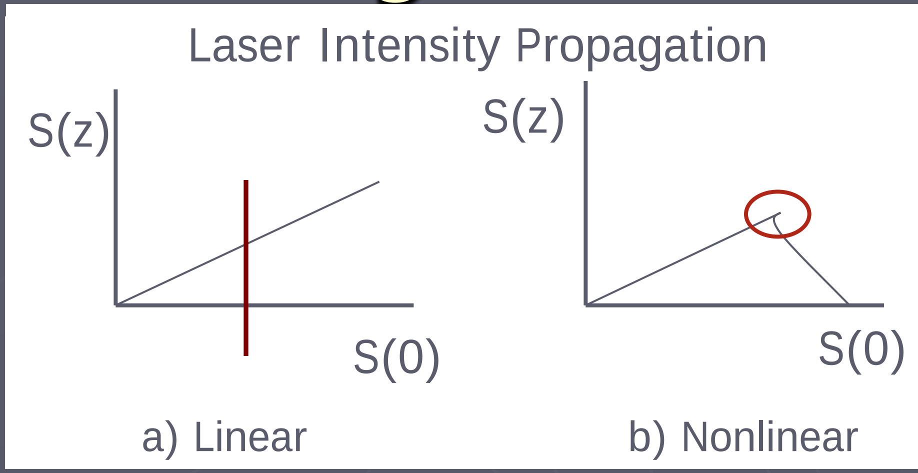


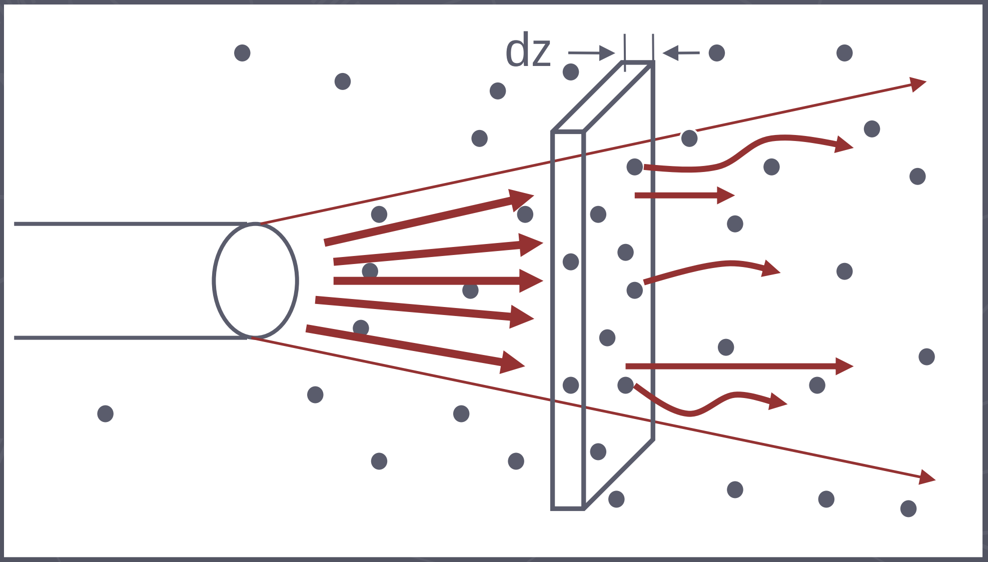





Thermal Blooming

- Temp ➡ Density ➡ Index of Ref.
- Induces Diverging lens
  - Optical Density > on edges
- Nonlinear Effect
- Maritime complication



Absorption and Scattering



$$\frac{dS}{dz} = -K(z)S$$

S(z) = Target Intensity

S(0) = Source Intensity

z = Propagation distance

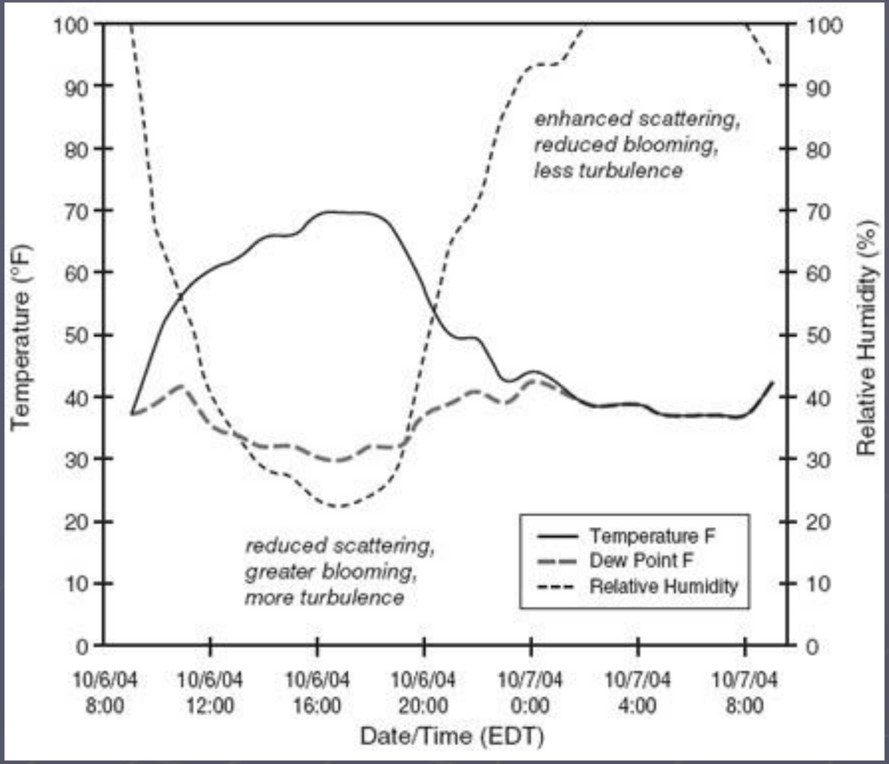
K = Attenuation Coefficient

Thermal Blooming

- Thermal Distortion factor, N_t
  - -models nonlinearity

$$N_t = -\frac{(dn/dT)}{n\rho c_p} \times \frac{KSZ^2}{vw}$$

  - dn/dT – slope of index of ref.
  - n – index of refraction
  - C_p – heat capacity (J/gm K)
  - P – density (gm/cm³)
  - K – absorption coefficient
  - S – laser intensity (W/cm²)
  - Z – range
  - w – beam radius
  - v – wind velocity
- Eqn shows that increasing S to counter absorption/divergence losses increases N_t



Non-linear and linear effects responding to changes in temperature, dew point, and relative humidity over a specified period (Narcisse et al., 2009).

Thermal Blooming PDE's

Mark Schmitt PDE

$$\nabla_{\perp}^{2}E(x,t)+2ink_{vac}\frac{\partial E}{\partial z}=-2nk_{vac}^{2}n_{2}IE+nk_{vac}k''\frac{\partial^{2}E}{\partial t^{2}}+(1-i\frac{\upsilon}{\omega_{0}})\frac{\omega_{p}^{2}}{c^{2}}E-i\beta^{(K)}I^{K-1}E$$

- Non-linear
- I = Intensity, E = Electric Field

$$I=\left(\frac{c|E|^{2}}{8\times 10^{7}\pi}\right)$$

NRL's HELCAP PDE, P. Sprangle et al.

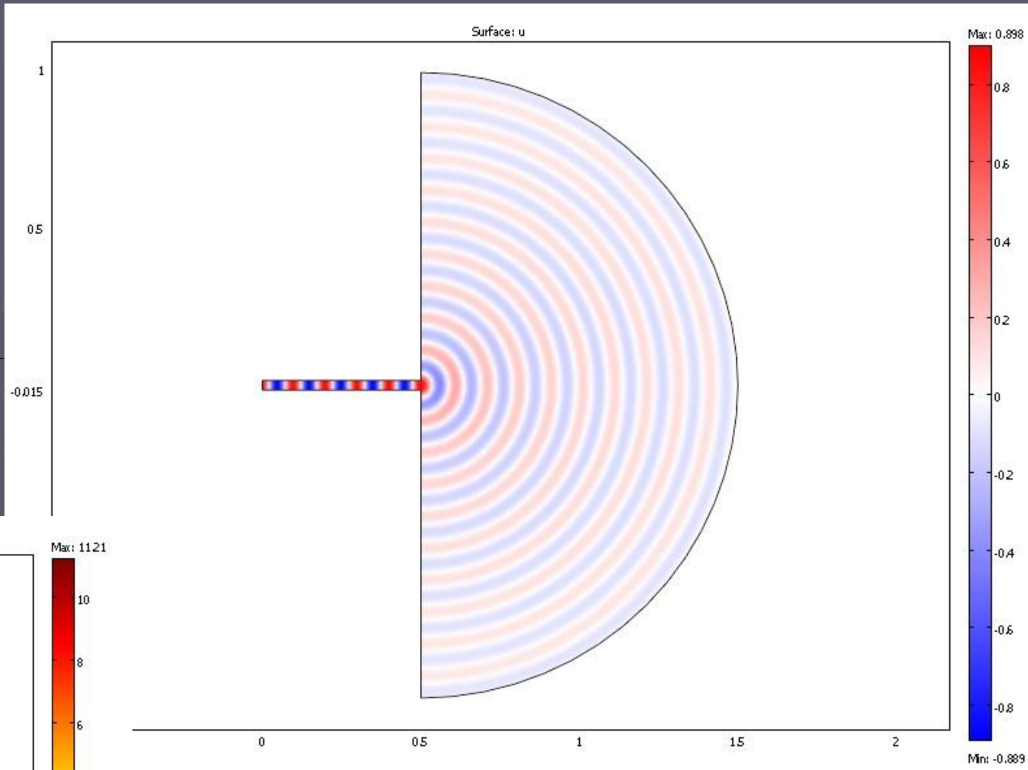
$$\frac{\partial A}{\partial z}=\frac{1}{2}\left[\frac{i}{k_{0}}\nabla_{\perp}^{2}-2ik_{0}(\partial n_{TB}+\partial n_{T}+\partial n_{A})-\alpha-\beta\right]A$$

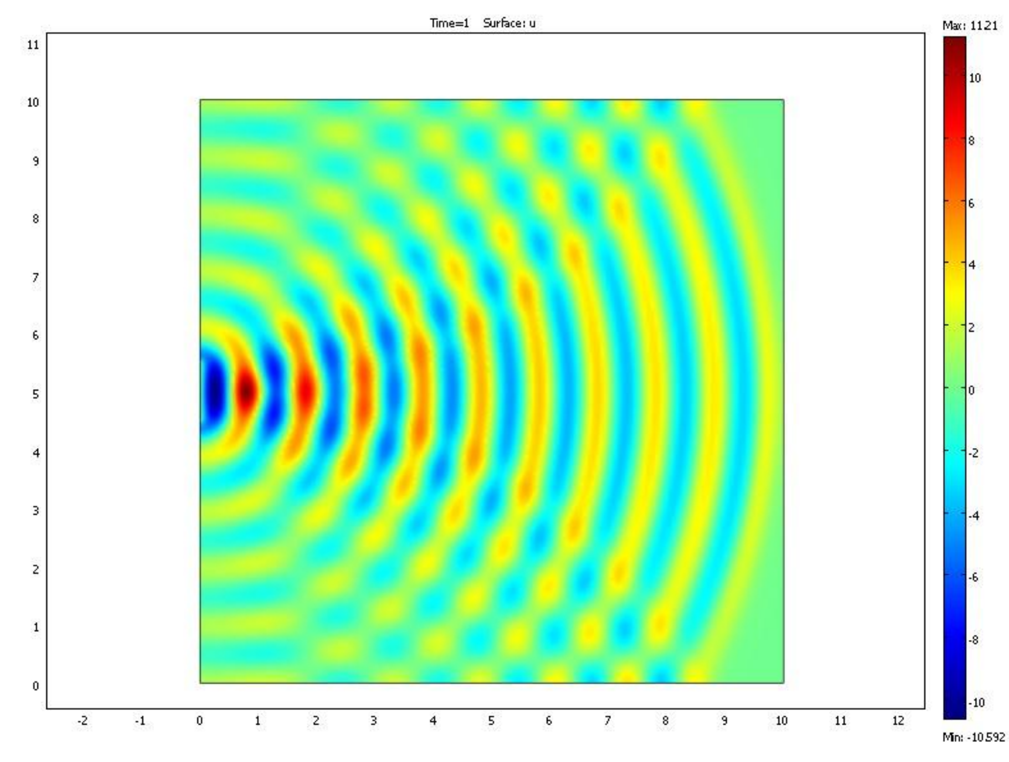
- Change in index of refraction due to thermal blooming, turbulence, and aerosol absorption dependent on amplitude

Diffraction in COMSOL

PDE, Coefficient Form

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c\nabla u - \alpha u + \gamma)$$
$$+ \beta \nabla u + a u = f$$





The Wave Equation

$$\frac{\partial^2 u}{\partial t^2} - \nabla \cdot (c\nabla u) = f(x, y, u, t)$$

Boundaries: Neumann

-Inflow: q = i*k, g = 2*i*k

-Outflow: q = i*k, g = 0

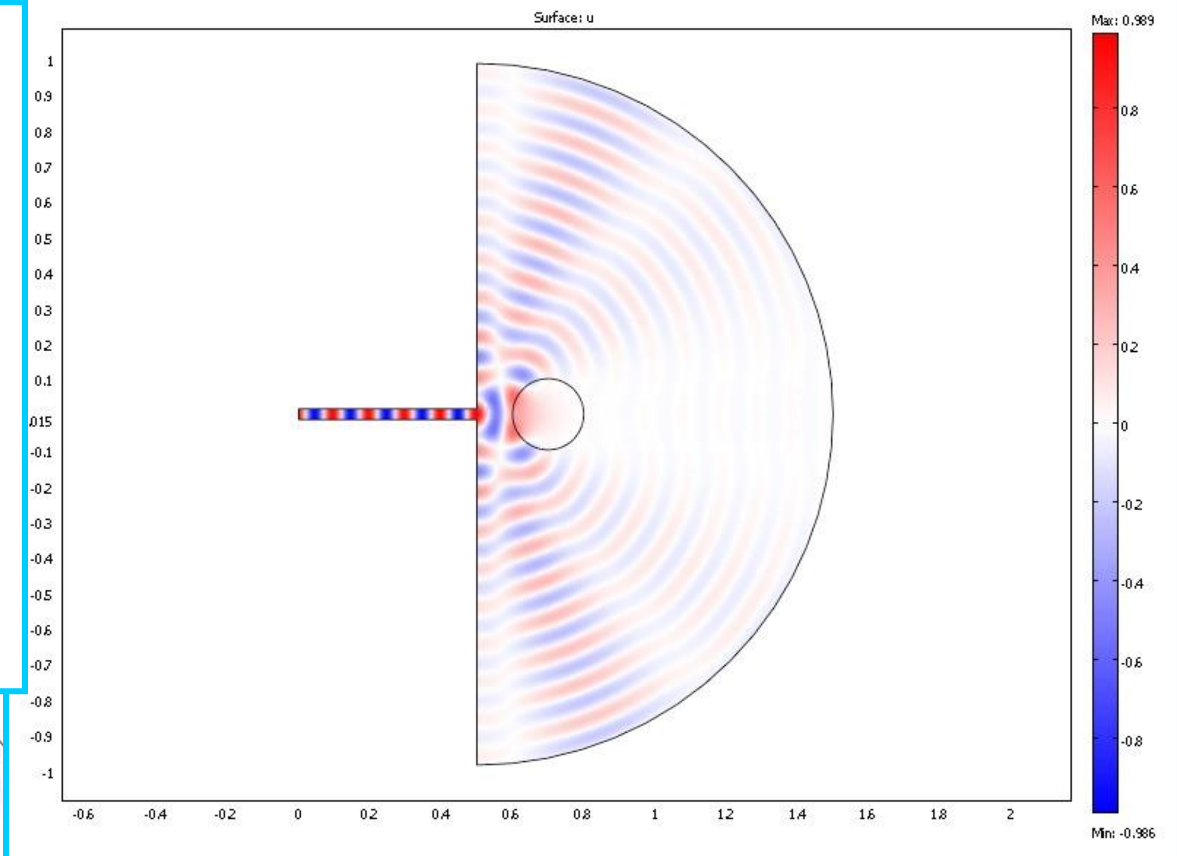
-Circle: q = 0, g = 0

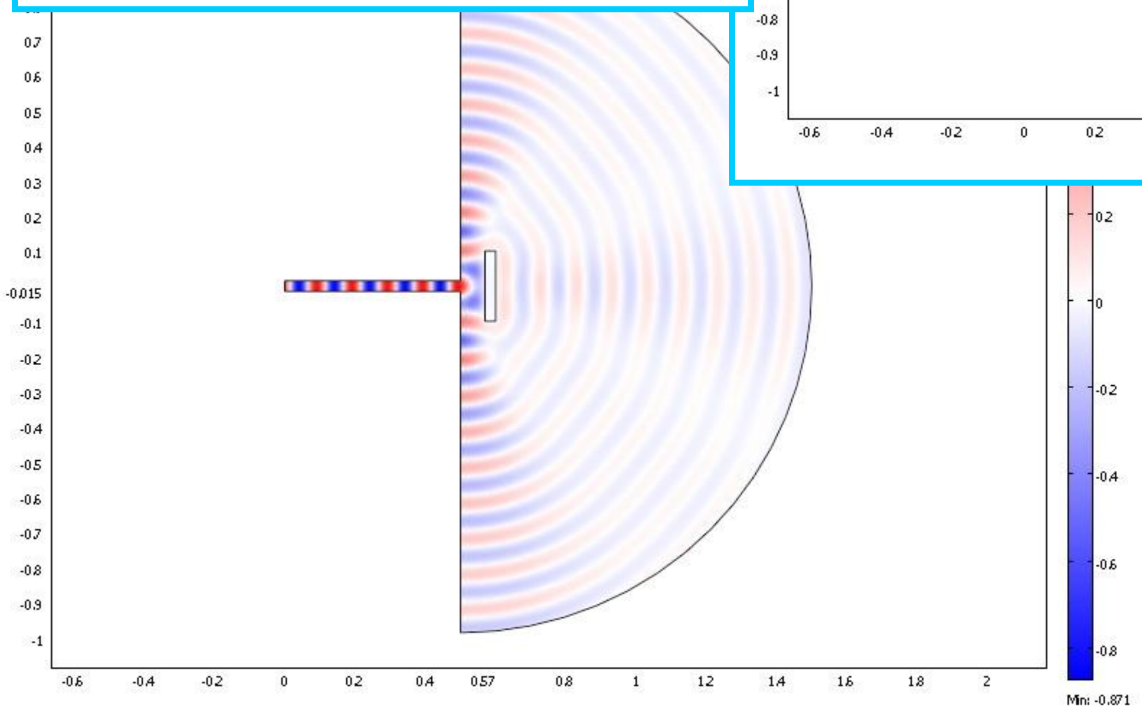
Subdomains: Disk

c = 1, diffusion coeff.

a = 0, absorption coeff.

f = 0, source term





-Outflow: q = i*k, g = 0

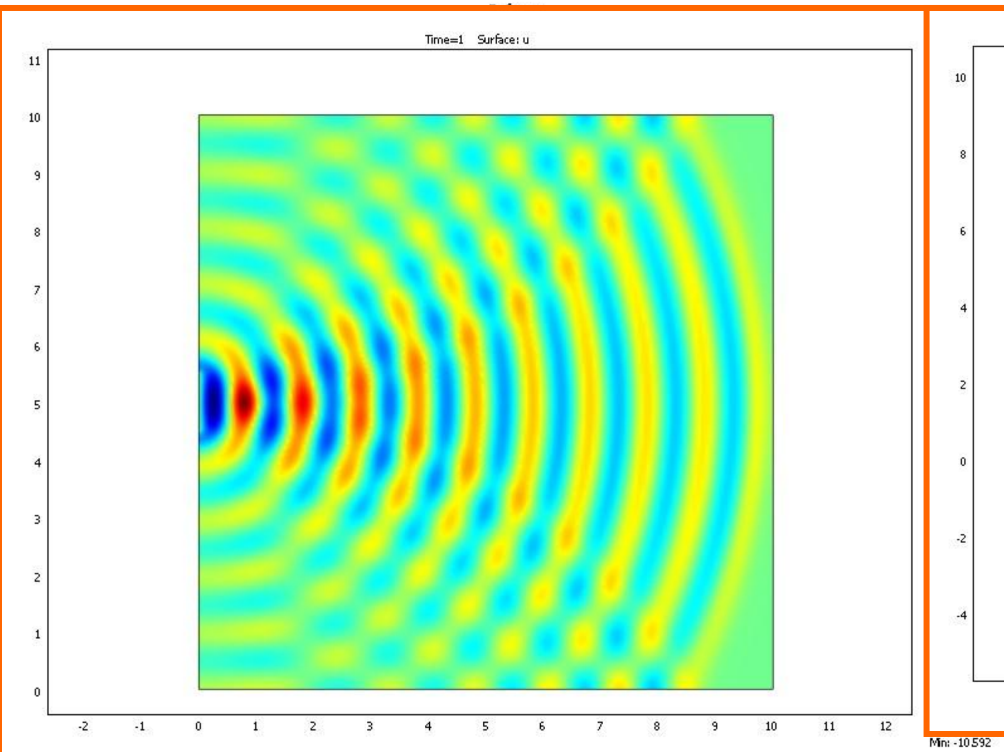
-Rectangle: q = 0, g = 0

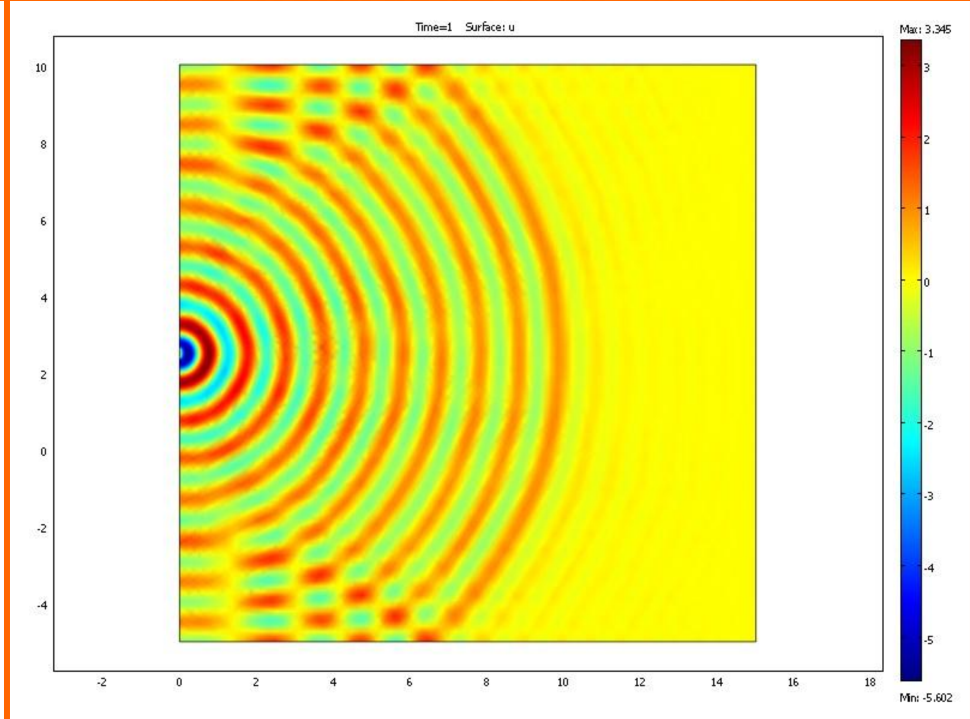
Subdomains: Rectangle

c = 100, diffusion coeff.

a = $-k^2$, absorption coeff.

f = 0, source term





Boundaries:

-Inflow: Dirichlet,

q = 0, g = 0, h = 1, r = 10sin(20πt)

-Outflow: Neumann, q = 0, g = 0

Subdomains:

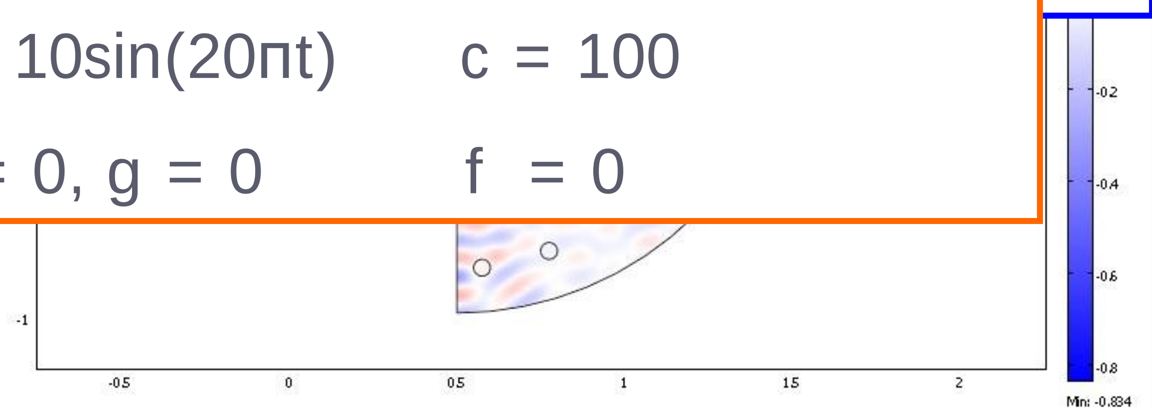
$e_a = 1$

c = 100

f = 0

a = 0, absorption coeff.

f = 0, source term



Future Work

- Enter Paraxial Wave Equation into COMSOL using the "PDE, General Form"

- Model the Mark Schmitt EQN with COMSOL using the "PDE, General Form"

$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot \Gamma = f$$

- Mark Schmitt EQN

$$\nabla_\perp^2 E(x,t) + 2ink_{vac}\frac{\partial E}{\partial z} = -2nk_{vac}^2 n_2 IE + nk_{vac}k''\frac{\partial^2 E}{\partial t^2} + (1 - i\frac{\upsilon}{\omega_0})\frac{\omega_p^2}{c^2}E - i\beta^{(K)}I^{K-1}E$$

$$I = \left(\frac{c|E|^2}{8\times 10^7 \pi}\right)$$

$$\frac{\partial^2 E}{dt^2} = \nabla\cdot(c\nabla E) \Rightarrow \nabla^2 E + k^2 E = 0 \Rightarrow E(x,y,z) = e^{ikz}V(x,y,z)$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2}e^{ikz} + \frac{\partial^2 V}{\partial y^2}e^{ikz} + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}e^{ikz} + ikVe^{ikz}\right) + k^2 e^{ikz}V = 0$$

$$\Rightarrow e^{ikz}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right) + \frac{\partial^2 V}{\partial z^2}e^{ikz} + \frac{\partial V}{\partial z}(ik)e^{ikz} + ik\frac{\partial V}{\partial z}e^{ikz} - k^2Ve^{ikz} + k^2Ve^{ikz} = 0$$

$$\Rightarrow e^{ikz}\Delta_\perp V + \frac{\partial^2 V}{\partial z^2}e^{ikz} + 2ike^{ikz}\frac{\partial V}{\partial z} = 0 \Rightarrow \Delta_\perp V + 2ik\frac{\partial V}{\partial z} = 0$$

PWE

sources

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