Analysis of Burning Candle

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Abstract: The physics of a burning candle analyzed using COMSOL Multiphysics 3.5a are combined with generalized strategies to predict the heat transfer and fluid flow behavior of a burning candle. Heat transfer is incorporated through conduction, convection and radiation components. The low melting point of the candle wax leads to a phase change that allows mass transport via capillary flow prior to combustion in the flame. The analyses have been used to predict the velocity flow field for a three-wick candle contained within a ceramic vessel, spreading of the air flow away from the flame and temperature distributions around the burning candle. Predicted temperature distributions in the wax and candle container compare favorably with experimental measurements.

Keywords: Radiation, Conduction, Convection, Artificial diffusion, Heat transfer

1. Introduction

A burning candle, providing both heat and light, simultaneously represents one of the oldest technologies still in use today and incorporates one of the most complex multiphysics environments. Despite widespread use over many centuries it was not until over 150 years ago that Faraday performed the first comprehensive scientific study on the physics of candle burning (1). Through a series of simple and elegant experiments, Faraday provided exceptional insight into the chemical structure and fluid mechanics associated with the burning candle. Combustion of liquid wax transported by capillary flow through the porous wick results in a flame with a highly non-linear temperature profile in which local temperatures in the candle flame can exceed 1400 °C. Heat transfer from the candle flame occurs by the combined processes of conduction, convection and radiation and produces a solid to liquid phase change in the low melting point wax.

This work combined COMSOL Multiphysics with generalized strategies to analyze the heat transfer and fluid flow taking place due to candle burning. The results of these analyses were compared with experimental data.

2. Analysis

The goal of this modeling effort was to predict air velocities and temperatures in the vicinity of a candle flame without resorting to excessive computational resources and requiring excessive simulation time. To this end, the following simplifications were incorporated:

1) The heat generated in the candle flame by the combustion of the wax was approximated by a simple heat source.
2) The problem was treated as being stationary.
3) For a three-wick candle, one-sixth symmetry was assumed.

Within the framework of these simplifications, heat transfer by means of conduction, convection, and radiation was modeled in the candle and surrounding regions. In the fluid domains, the physics are described by the conservation of mass, momentum, and energy according to the following equations:

$$\nabla \cdot \left(\rho \ \mathbf{u}\right) = 0$$

$$\rho \ \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nabla \cdot \left(\eta\left(\nabla \mathbf{u} + \left(\nabla \mathbf{u}\right)^T\right) - \frac{2}{3}\eta\left(\nabla \cdot \mathbf{u}\right)\mathbf{I} \right) + \rho \ \mathbf{g}$$

$$\nabla \cdot \left(-k\nabla T\right) = Q - \rho \ c_p \mathbf{u}$$

The viscous heating and pressure work terms neglected in the energy equation. In the above equations, ρ is the density, $\mathbf{u}$ is the velocity vector, p is the pressure, η is the dynamic viscosity, $\mathbf{g}$ is the gravitational acceleration vector, k is the thermal conductivity, T is the temperature, Q is a heat source term, and c_p is the specific heat capacity. The viscosity, thermal conductivity, and specific heat capacity are functions of temperature, while the density is a function of both temperature and pressure. Since the spreading of the plume cannot be fully accounted for in a stationary, symmetric model, artificial diffusion was added in the plume region to give an approximation of this behavior. The appropriate amount of artificial diffusion was determined by means of empirical measurements.

In the solid domains, conduction was modeled using the simple heat equation:

$$\nabla \cdot (-k\nabla T) = Q$$

where the source term Q includes latent heat effects due to the melting and evaporation of the wax.

Radiative heat transfer from the flame to the surrounding objects was included and the heat flux at a surface due to radiation was modeled as:

$$q_r = \varepsilon\left(G_m + F_{amb}\sigma\, T_{amb}^4 - \sigma\, T^4\right)$$

where ε is the emissivity of the surface, G_m is the mutual irradiation from other surfaces, F_{amb} is the ambient view factor, σ is the Stefan-Boltzmann constant, T_{amb} is the far-away ambient temperature, and T is the temperature at the surface. G_m is a function of the radiosity, which is the sum of the emitted heat flux and the reflected heat flux and is given by the following equation:

$$J = (1-\varepsilon)\left(G_m + F_{amb}\sigma\, T_{amb}^4\right) + \varepsilon\sigma\, T^4$$

Since COMSOL 3.5a supports only surface-to-surface radiation, radiation from the flame region was enabled by creating a flame radiation boundary that is non-locally coupled to the radiating gas volume. The radiosity on this boundary is given by the equation:

$$J = \varepsilon\sigma\, \left(T^4\right)_{avg}$$

where $(T^4)_{avg}$ is the average value of T^4 in the radiating gas volume. The emissivity for the radiating flame surface was chosen so that the ratio of radiated power to total power matched empirical measurements (2). The heat flux due to radiation, q_r, was set to zero at this boundary, while the cooling in the flame region due to radiation was accounted for in the radiating gas volume by means of a source term:

$$Q = -\frac{\varepsilon\sigma\, T^4 A}{V}$$

where A is the area of the flame radiation boundary and V is the volume of the radiating gas. Thus, the total power loss in the radiating gas is equal to the total power radiated from the flame boundary.

3. Results

Validation of the analytical approach was made by comparing predictions of the temperature with those measured at various locations. A series of temperature measurements were made to identify the temperature as a function of candle height, location along the top surface of the wax and at the location of the solid-liquid interface. An example of the comparison obtained between predicted and experimental data along the top surface of the wax is provided in Figure 1.

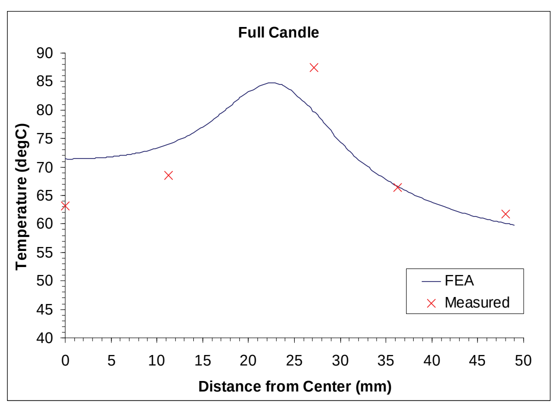

Figure 1. Comparison of measured and predicted temperature profile along the top surface of the wax through the wick.

Predictions of the temperature of the ceramic container also show good agreement with the experimental measurements, Table 1.

Table 1: Measured vs Predicted temperatures

	Measured (degC)	**FEA (degC)**
Full Candle	40	46
Half Candle	44	49

The solution methodologies developed during this work can predict the flow and temperature distribution associated with a burning candle flame. The effect of candle height on the velocity field associated with a burning candle within the container can be seen in Figures 2 and 3.

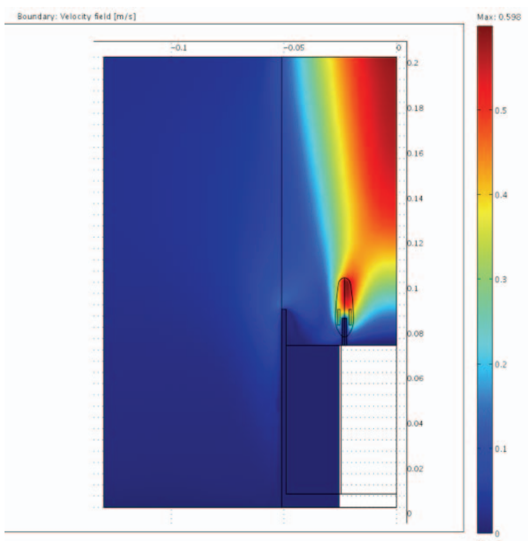

Figure 2. Velocity field within the plume for a full candle.

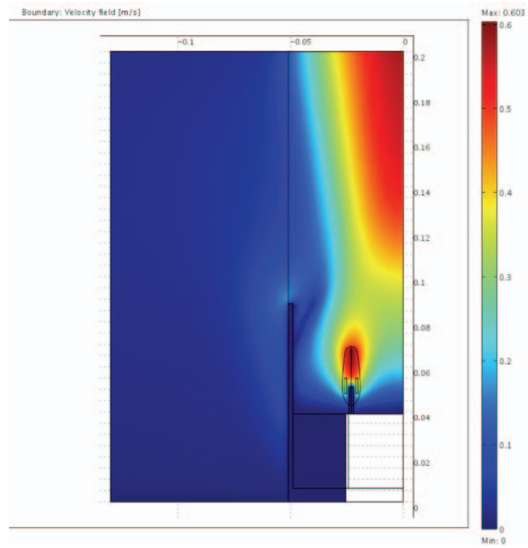

Figure 3. Velocity field within the plume of a burning candle flame for a half full candle.

Generally, the plume is concentrated towards the center of the container but the precise shape of the plume is also influenced by the horizontal location of the flame, see Figures 4 and 5.

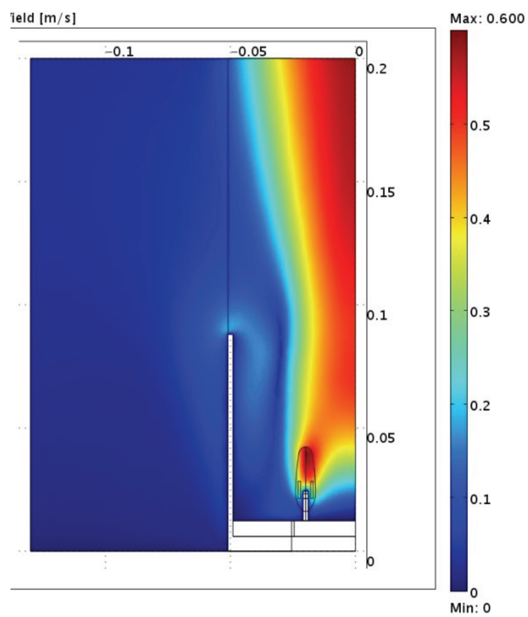

Figure 4. Velocity field within the plume of a burning candle flame when flame is close to bottom of the container and located towards center of container.

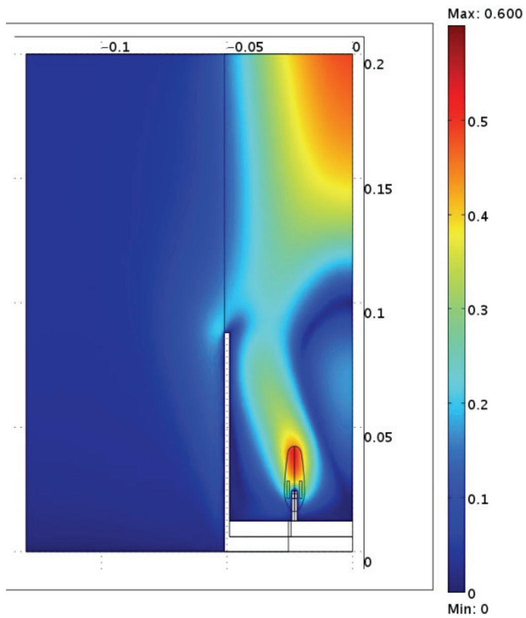

Figure 5. Velocity field within the plume of a burning candle flame when flame is close to bottom of the container and located 1/8" towards container walls.

As the height of the wax in the container decreases, the sides of the container constrain expansion of the plume, forcing it to retain a concentration towards the center. Similarly, as the position of the flame moves towards the container walls in the horizontal plane the sides of the container act to constrain plume expansion beyond the container walls.

Convective flow and heat transfer outside the immediate vicinity of the candle flame provides the transfer of heat to the nearby surroundings, Figure 4.

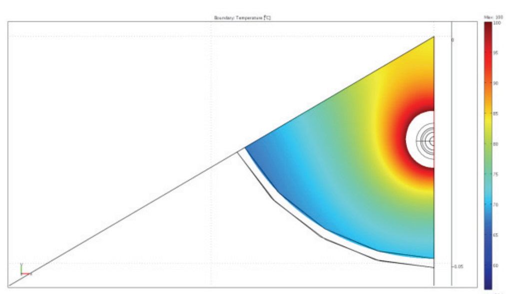

Figure 4. Temperature distribution away from candle flame along top of wax surface.

4. Conclusions

A computational model of a burning candle has been developed that incorporates heat transfer through conduction, convection and radiation. Melting of the candle wax leads to a phase change that allows mass transport via capillary flow prior to combustion in the flame. Predictions of the velocity flow field for a three-wick candle contained within a ceramic vessel, demonstrate spreading of the air flow away from the flame and temperature distributions around the burning candle.

5. References

1. Faraday, M., Christmas Lectures, Royal Institution, 1860.
2. Hamins, A., Bundy, M., Dillon, S.E., "Characterization of Candle Flame", *Journal Fire Protection Engineering*, V15, p265-285, November 2005.