Dynamic Structural Modelling of Wind Turbines Using COMSOL Multiphysics

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Abstract: This paper presents a study of a wind turbine subjected to wind and seismic loading, carried out using COMSOL Multiphysics. The dynamic properties and response of wind turbine structures are of interest, as recent developments in wind energy have led to the design and construction of increasingly large and flexible turbine structures. A typical turbine structure model was created in COMSOL and transient simulations were carried out under wind and earthquake excitations. It was found that the structural responses due to seismic loads have significantly different characteristics from those due to wind loads, and warrant further study.

Keywords: Structures, Dynamics, Turbines, Wind, Seismic.

1. Introduction

Growing interest in wind energy has led to the design and construction of increasingly large and flexible wind turbines, with rotor diameters as large as 120 m. Increasing sizes and power ratings for wind turbines lead naturally to increased structural loading, as well as increased concern regarding seismic hazard.

The primary loadings on a wind turbine are dynamic, namely wind and seismic loads. These loads are stochastic processes containing excitation components in a broad range of frequencies. Consequently, accurate estimation of the structure's natural frequencies of vibration is especially important, so that the effects of structural resonance can be avoided or reduced.

This paper will discuss the nature of wind turbines and their loading, give the equations governing their behaviour, present an example turbine model in COMSOL, summarize the results, and suggest further research.

2. Wind Turbine Structures and Loading

The design and analysis of wind turbine structures may be based on several applicable codes. This paper will primarily discuss those set forth in International Electrotechnical Commission (IEC) 61400-1, "Wind turbines. Part 1: Design requirements" [1]. The following sections will discuss the nature of wind turbine structures, wind loading on turbine structures and seismic loading on turbine structures.

2.1 Wind Turbine Structures

An industrial-class wind turbine typically consists of three blades connected by a rigid hub. The hub is attached to a nacelle containing the mechanical and electrical components of the turbine. The blade-hub-nacelle assembly is supported by the tower. A side-profile sketch of a typical wind turbine can be seen in Figure 1.

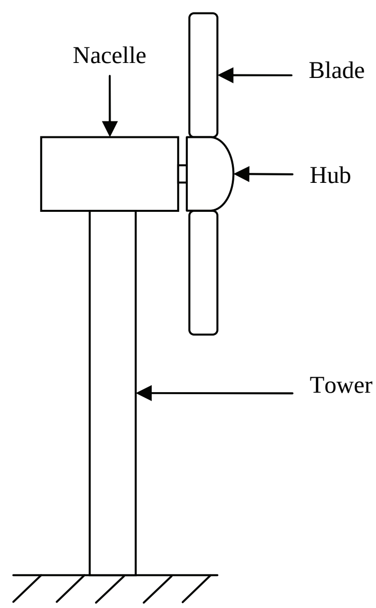

Figure 1. Side-profile view of a typical wind turbine indicating main components.

A wind turbine is classified as a vertical cantilever structure, the tower, supporting three cantilever sub-structures, the blades. Since the blades and tower are long compared to their cross-sectional dimensions, they are assumed to be Euler-Bernoulli beams, with dominant response contributed by the flexural modes. If the masses of the nacelle and hub are assumed to be lumped at the top of the tower, the turbine can be modelled as shown in Figure 2.

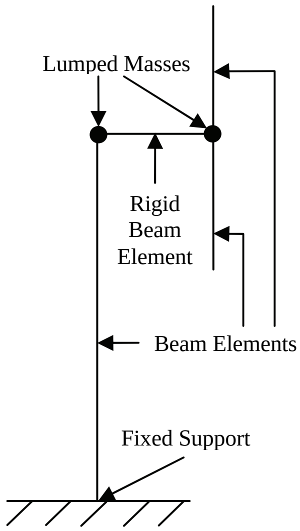

Figure 2. Structural idealization of wind turbine.

The centrifugal stiffening effect on the blades due to rotation is ignored for simplicity. Interested readers are referred to work by Naguleswaran [2] and by Hansen [3] for details.

The tower and blade sections may have constant or varying mass and stiffness distributions. Commonly, the tower is a cylindrical steel shell which tapers in diameter and thickness, while the blades are both tapered and twisted from root to tip and are constructed from glass-fibre-reinforced plastic (GRP).

2.2 Wind Loading on Wind Turbines

The primary loading on wind turbines is of a stochastic nature. Instantaneous wind speed and pressure at a fixed point in space are random signals. They can be simulated using Power Spectral Density (PSD) functions, which have been developed in the literature, such as [4].

The PSD function is a measure of the frequency content of a signal, in this case wind speed. Given a PSD function for turbulent wind speed, a time-history can be simulated using random phase data. IEC 61400-1 specifies the use of the Mann uniform shear turbulence model or the simpler Kaimal spectrum. The Kaimal spectrum is presented in IEC 61400-1 as

$$PSD(f) = \frac{4\sigma^2 \frac{L}{V_{hub}}}{\left(1+6\frac{fL}{V_{hub}}\right)^{\frac{5}{3}}} \tag{1}$$

where σ is the standard deviation of wind speed, L is a length scale, V_{hub} is the wind speed at hub height, and f is the frequency in Hz.

Coherence between wind time histories at two points is represented by a coherence function, typically of a form such as

$$Coh(r,f) = \ exp\left[-12\left(\left(\frac{fr}{V_{hub}}\right)^2 + \left(\frac{0.12r}{L}\right)^2\right)^{0.5}\right] \tag{2}$$

where r is the separation distance. The value of coherence is 1 when the separation distance is zero, and decreases as the separation distance increases. The coherence decreases more quickly at higher frequencies, as higher-frequency turbulence is caused by smaller, localized atmospheric eddies.

Wind loading on turbines is a function of the wind pressure at each point on the structure, as well as the geometry. As wind passes over the blade, it creates lift and drag forces, which can be resolved into components perpendicular and parallel to the oncoming wind direction. The perpendicular component is the force which causes the blades to rotate, while the parallel component is a structural load on the turbine.

2.3 Seismic Loading on Wind Turbines

Seismic loading on wind turbines is an area which has not been studied in depth. IEC 61400-1 does not provide explicit guidance for seismic design, since in most locations wind loads will govern over seismic loads. However, interest in wind energy is increasing in areas of higher seismic hazard, including California, the West Coast of Canada, and some parts of India, among others. This warrants a re-visit of the issue.

IEC 61400-1 states that assessment of earthquake conditions should use ground acceleration and response spectrum methods as defined in local codes. Annex C of the standard provides a simplified approach, in which the mass of the hub, blades, nacelle, and half of the mass of the tower are lumped at the top of the tower. This lumped mass is subject to the design acceleration in the first tower-bending mode, as determined from local response spectra. If the turbine can withstand these loads, applied in conjunction with emergency stopping loads, then the design is considered adequate.

Alternatively, many seismically sensitive structures are subjected explicitly to normalized ground acceleration time-histories which were either recorded or synthesized from known PSD or response spectrum functions. A method of constructing artificial time histories is outlined in [5]. This method is implemented in the GH Bladed analysis and design program.

3. Structural Dynamics Formulation

3.1 Governing Equations

The equation of motion for an Euler beam is

$$m(x)\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u}{\partial x^2}\right]=p(x,t) \qquad (3)$$

where *m(x)* is the mass density per unit length of the beam, *EI(x)* is its flexural stiffness, *u(x,t)* is its transverse displacement, and *p(x,t)* is the external load.

The displacement *u(x)* can be represented as

$$u(x,t)=\sum_{i=1}^{\infty}\varphi_i(x)q_i(t) \qquad (4)$$

where *φ(x)* are the mode shapes of the structure and *q(t)* are known as the modal coordinates.

The mode shapes are determined by solving

$$[EI(x)\varphi''(x)]''-\omega^2 m(x)\varphi(x)=0 \qquad (5)$$

where primes denote spatial derivatives. There are an infinite number of functions *φ(x)* and circular natural frequencies ω for the beam.

The modal coordinates, due to the orthogonality of the mode shapes with respect to mass and stiffness, are not coupled, and are determined by solving the equation

$$\boldsymbol{M_G}\ddot{q}+\boldsymbol{C_G}\dot{q}+\boldsymbol{K_G}q=\boldsymbol{F_G} \qquad (6)$$

Where overdots represent time derivatives and $\boldsymbol{M_G}$ and $\boldsymbol{K_G}$ are the *n x n* generalized mass and stiffness matrices, respectively:

$$M_{G\,i,j}=\int_0^L\varphi_i(x)m(x)\,\varphi_j(x)dx \qquad (7)$$

$$K_{G\,i,j}=\int_0^L\varphi_i''(x)EI(x)\,\varphi_j''(x)dx \qquad (8)$$

where *n* is the number of modes used to approximate the true solution. All off-diagonal terms are zero. If the mode shapes are assumed to be normalized such that the diagonal entries in the generalized mass matrix are equal to 1, then matrix $\boldsymbol{C_G}$, the generalized damping matrix, will be diagonal, with on-diagonal entries

$$C_{G_{i,i}}=2\zeta_i\omega_{ni} \qquad (9)$$

where ζ_i is the damping ratio for mode *i*, and ω_{ni} is the natural frequency for mode *i*. Typical damping ratios for a structure such as a wind turbine are in the range of 1% of critical damping. The generalized load vector, $\boldsymbol{F_G}$, is an *n x 1* vector with time-dependent entries

$$F_{G\,i,1}=\int_0^L\varphi_i(x)p(x,t)dx \qquad (10)$$

3.2 Finite Element Formulation

The structure may be divided into multiple beam elements, with local mass and stiffness matrices for each element derived using cubic polynomial shape functions, similarly to the equations (7) and (8), with the shape functions replacing the mode shapes. The local matrices are assembled into global mass and stiffness matrices, $\boldsymbol{M}$ and $\boldsymbol{K}$, respectively. Solving the eigenvalue problem

$$\boldsymbol{K}-\omega_n^2\boldsymbol{M}=0 \qquad (11)$$

yields the natural frequencies and mode shapes.

The generalized mass and stiffness matrices can be expressed as

$$\boldsymbol{M_G}=\boldsymbol{\varphi^T M\varphi} \qquad (12)$$

$$\boldsymbol{K_G}=\boldsymbol{\varphi^T K\varphi} \qquad (13)$$

where **φ** is a matrix whose columns are the eigenvectors of the structure. The generalized damping matrix can be expressed as

$$\boldsymbol{C_G}=\boldsymbol{\varphi^T C\varphi} \qquad (14)$$

where **C** is a linear combination of **M** and **K**. This is known as Rayleigh damping, with the coefficients for **M** and **K** determined by assuming that the damping ratio is known in two modes. The load vector can be expressed as

$$\boldsymbol{F_G} = \boldsymbol{\varphi^T P} \tag{15}$$

where **P** is a vector of nodal loads

The generalized matrices in equations (12) to (14) and the load vector are used to solve for the modal coordinates, which are transformed into displacements.

3.3. Wind Loading

Wind loads are calculated using the equation

$$F_W = \frac{\rho C_D U^2 A}{2} \tag{16}$$

where F_W is the total wind force on a given area, ρ is the density of air, C_D is a drag coefficient, U is the wind speed, and A is the exposed area.

Generation of wind speed time-histories incorporating PSD and coherence functions is discussed in [3]. A linear set of time histories can be simulated to act on the tower, while a radial grid can be simulated to act on the blades, as shown in Figure 3.

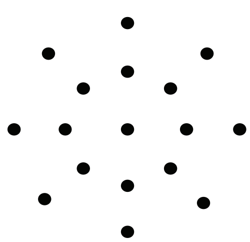

Figure 3. Radial grid of time histories.

The number of points along the blade and the angular division can be varied as desired. The blades are assumed to sweep through each radial line and be subjected to what is effectively a 'frozen' turbulence field. The wind forces determined from the nodal wind speeds are applied as nodal loads to the structure.

3.4 Seismic Loading

Loading due to ground acceleration is applied as an effective load equal to

$$p_{eff}(x, y, z, t) = -m(x, y, z) a_g \tag{17}$$

where p_{eff} is the effective load, m is the mass distribution, and a_g is the ground acceleration. The negative sign indicates that the load is applied as effective inertial forces.

The effective load can be transformed into time-varying nodal loads as discussed for wind loading and applied to the finite element model.

3.5 Approximate Solutions

If the blades are assumed to vibrate at frequencies far from those at which the tower would vibrate if the blades were rigid, the blades and tower can be considered to be separate systems, with the blades treated as cantilevers and the tower treated as a vertical cantilever with a lumped mass and inertia at its head.

For the blades, traditional methods such as those in [6] can be applied. The properties of a cantilever with a head mass and inertia were studied in [7]. For a cantilever of length L with uniform mass m and stiffness EI per unit length, carrying a head mass of M and inertia of J, the natural frequencies can be found by solving

$$\begin{aligned}&\left[1 - (\beta L)^4 R_M R_J\right](\cosh(\beta L)\cos(\beta L)) - \\ &\left[(\beta L) R_M + (\beta L)^3 R_J\right](\cosh(\beta L)\sin(\beta L)) + \\ &\left[(\beta L) R_M - (\beta L)^3 R_J\right](\cos(\beta L)\sinh(\beta L)) + \\ &\left[1 + (\beta L)^4 R_M R_J\right] = 0\end{aligned} \tag{18}$$

where R_M and R_J are known as mass and stiffness ratios, respectively, and given as

$$R_M = \frac{M}{mL} \tag{19}$$

$$R_J = \frac{J}{mL^3} \tag{20}$$

and the parameter β is given by

$$\beta^4 = \frac{\omega^2 m}{EI} \tag{21}$$

where ω is the circular natural frequency.

For a cantilever with non-uniform mass and stiffness, formulating the problem analytically is more difficult and finite element methods are a more efficient solution. However, these approximate solutions are valuable as checks on the behaviour of more detailed FE models.

4. Use of COMSOL Multiphysics

The implementation of the problem and solution in COMSOL is discussed below.

4.1 Structural Idealization

The wind turbine is idealized using the 3-D Euler Beam module in COMSOL, as shown in Figure 2. COMSOL's flexible definition of variables allows for simple modelling of structures having non-linear mass and stiffness along their lengths. Although the sample turbine discussed in this paper has uniform tower and blade sections, tapered or variable sections are easily modelled.

4.2 Application of Loads

The wind and seismic loads were applied as user-defined functions. The wind loads were imported into COMSOL as functions, each applied uniformly along a section of the blade.

The seismic loading was based on recorded earthquake ground accelerations retrieved from the NGA Database of the Pacific Earthquake Engineering Research (PEER) Centre. A single component of ground acceleration was applied in the direction of the applied wind loads. The effective inertial loads were applied as in Section 3.4 of this paper. COMSOL uses direct solution of the equations of motion rather than the modal superposition method discussed in Section 3.

5. Turbine Simulations

The example turbine studied in this paper is based on the one presented by Murtagh et al [8]. The turbine has a 60 m hub height. The tower is a uniform cylindrical shell with a 3 m outer diameter and 15 mm wall thickness. It is built from steel with a Young's Modulus of 210 GPa and mass density of 7850 kg/m^3. The tower has a drag coefficient of 1.2. The blades are 30 m long uniform hollow rectangular sections with outer width of 2.8 m, outer depth of 0.8 m, and wall thickness of 10 mm. The outside width dimension is parallel to the rotor plane. The blades have a Young's Modulus of 650 GPa, a mass density of 2100 kg/m^3, and a drag coefficient of 2.0. It is assumed that the offset between the tower and the rotor plane is 4 m, and that the rigid offset from the hub to the base of the blade is 3 m. The combined mass of the nacelle and hub is approximately 20,000 kg, 1/3 of which is lumped at the hub and 2/3 of which is lumped at the head of the tower. The COMSOL model is shown in Figure 4.

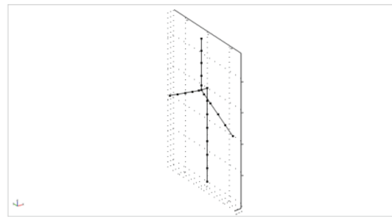

Figure 4. COMSOL model of wind turbine.

As shown, the model was built with one blade in the vertical position. This is equivalent to subjecting the turbine to loading while it is in a parked position. The effects of rotating blades are neglected in this paper.

The mean wind speed at hub height was assumed to be 20 m/s with a 2 m/s standard deviation. The wind PSD and coherence differ from those used in [8]. The Kaimal spectrum and coherence functions are used as defined in [1], presented in Section 2.2 of this paper. In [8], some of the energy in the PSD function was redistributed to account for the effects of the rotating blades. In this paper, it is assumed that the PSD is the same at all points on the turbine. Coherent wind speeds were generated at 3 points along the blade, spaced 120° apart, allowing the wind loads to be applied directly to the blades rather than having the blades sweep through a set of time-histories. This simplification will be a subject of future research.

A time-history of the 1992 Cape Mendocino earthquake, recorded at Station CDMG 89156 Petrolia, record CAPEMEND/PET000, was applied as seismic loading. The two-component motion was scaled to a maximum resultant acceleration of 0.5g, giving this component of the time-history a peak acceleration of 0.41g.

6. Results and Discussion

The model was subjected to eigenfrequency analysis as well as transient analysis due to wind and seismic loading.

6.1 Mode Shapes and Natural Frequencies

The natural frequencies of the structure were investigated using COMSOL's Eigenfrequency solver. A summary of the first seven eigenfrequencies found is given in Table 1.

Table 1: Eigenfrequencies and mode descriptions

Frequency (Hz)	Description of Mode Shape
0.457	Tower bending across-wind
0.464	Tower bending along-wind
1.685	Tower torsion
2.495	Tower bending across-wind
2.608	Tower bending along-wind
4.204	Blade and tower bending
5.322	Blade and tower bending

The first two frequencies correspond well to those estimated by equation (18) for a uniform beam with a point mass and inertia. The higher frequencies, however, differ from those calculated by the approximate method, indicating that the higher modes include both tower and blade bending. Similarly, the natural frequency of the blades as cantilevers is approximately 5 Hz, fairly close to the first mode of blade bending, but the higher eigenfrequencies diverge from those predicted by this simplification.

Based on the eigenfrequencies calculated, Rayleigh damping was implemented with a damping ratio of 1% of critical at frequencies of 0.46 Hz and 5 Hz. This differs from the approach in [8], in which the blades and tower were formulated separately, with 1% damping in each of the first three uncoupled modes for both.

6.2 Response to Wind Loading

The structure was subjected to wind loading time histories simulated at 3 discrete points along the blades. Each blade was discretized into 3 sections. Each section was then subjected to a uniform and time-varying load computed from the time histories, with mean wind speeds of 20 m/s and standard deviations of 2 m/s. The wind loading on the tower was neglected.

The total load imposed on the blades can be seen in Figure 5.

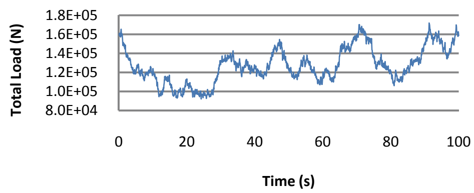

Figure 5. Total load applied to blades

The total load applied to the blades had a mean value of about 130 kN, and had a similar PSD to that used to generate the individual wind speeds. The along-wind displacement at hub height due to wind loading is shown in Figure 6.

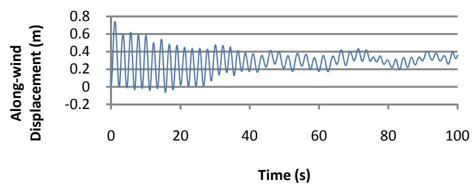

Figure 6. Head displacement under wind loading.

The variation in along-wind displacement was mostly at the first natural frequency of the tower, around 0.46 Hz. At-rest conditions were specified at time zero, so the response occurring after 40 seconds was approximately steady-state, as the effects of the initial conditions had been damped out. The base moment in the tower varied mostly at the first tower bending natural frequency as well, and had a time history qualitatively resembling Figure 6.

6.3 Response to Seismic Loading

The time-history of the Cape Mendocino earthquake is shown in Figure 7.

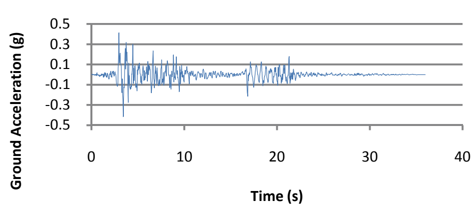

Figure 7. Cape Mendocino ground acceleration

The displacement of the tower head due to the ground acceleration is shown in Figure 8.

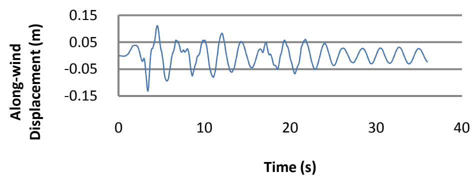

Figure 8. Head displacement under seismic load.

The head displacement consisted mostly of vibration at the first along-wind tower bending frequency. The base moment in the tower is shown in Figure 9.

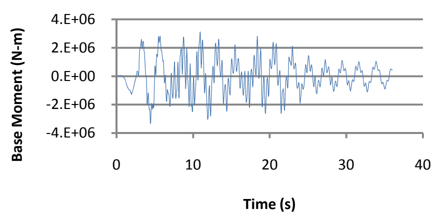

Figure 9. Base moment in the tower under seismic load.

As shown, the base moment contained more contributions from higher-frequency modes than did the displacement at the head.

7. Conclusions

This paper has summarized the basic structural idealization of a wind turbine and the wind and seismic loads applied to it, presented the governing equations, discussed the implementation of the problem in COMSOL, and provided a numerical example.

COMSOL is well-suited to dynamic structural analysis of wind turbines due to its flexible definition of constants and functions, user-friendly visualization, and efficient solvers.

Analysis of the sample turbine showed that although the displacement and base moment due to wind were greater in magnitude than those due to ground acceleration, the responses due to ground acceleration contained contributions from different modes and warrant further study.

8. Future Work Using COMSOL

The example turbine had uniform sections for both tower and blades. As discussed above, COMSOL allows for significant flexibility in definition of mass and stiffness properties, and could easily accommodate more complicated tapered and non-uniform sections.

The example turbine presented included wind loads based solely on drag. A real wind turbine blade is tapered and twisted, and the along-wind loading on the turbine is based on both the lift and drag coefficients of the blade. The equations governing wind loads could be implemented in a COMSOL model with realistic blade geometries.

Finally, future research may involve the use of the MATLAB interface to generate wind and seismic loading in a more efficient way as well as to perform extensive parametric studies.

9. References

1. International Electrotechnical Commission, IEC 61400-1, "Wind turbines. Part 1: Design requirements" (2005)

2. S. Naguleswaran, Lateral Vibration of a Centrifugally Tensioned Uniform Euler-Bernoulli Beam, *Journal of Sound and Vibration*, **176**, 613-624 (1994)

3. Martin O.L. Hansen, *Aerodynamics of Wind Turbines*, 121-124. Earthscan, London (2008)

4. T. von Karman, Progress in the statistical theory of turbulence, *Proceedings of the National Academy of Sciences*, **34**, 530-539 (1948)

5. D. Witcher, Seismic Analysis of Wind Turbines in the Time Domain, *Wind Energy*, **8**, 81-91 (2005)

6. A.K. Chopra, *Dynamics of Structures*, *3rd Ed.*, 635-636. Pearson Prentice Hall, Upper Saddle River, NJ (2007)

7. P.J. Murtagh, B. Basu and B.M. Broderick, Simple models for natural frequencies and mode shapes of towers supporting utilities, *Computers and Structures*, **82**, 1745-1750 (2004)

8. P.J. Murtagh, B. Basu and B.M. Broderick, Along-wind response of a wind turbine tower with blade coupling subjected to rotationally sampled wind loading, *Engineering Structures*, **27**, 1209-1219 (2005)

10. Acknowledgements

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