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Dynamic Structural Modelling of Wind Turbines Using COMSOL Multiphysics

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Presentation Summary

- Introduction
- Wind Turbine Structures and Loading
- Structural Dynamics Formulation
- Use of COMSOL Multiphysics
- Turbine Simulations
- Results and Discussion
- Conclusions
- Future Work Using COMSOL

Introduction

Introduction

- Growing interest in wind energy is driving an increase in the size and power ratings of wind turbines
- Shown below is the Enercon E-126
  - Hub height -138 m
  - Rotor diameter - 126 m
  - Rated power - 6 MW

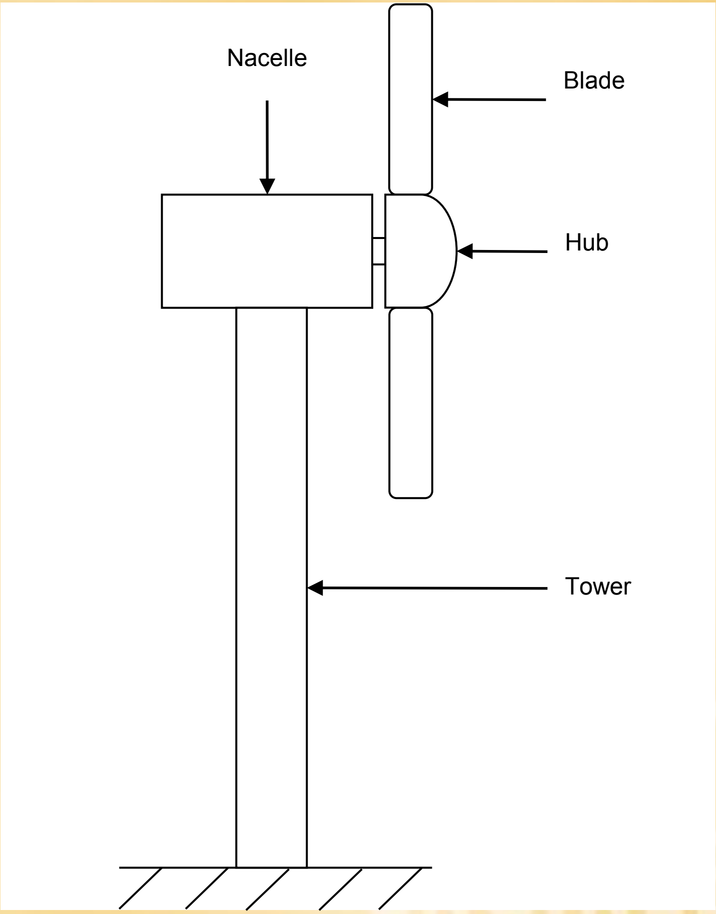
Introduction – Research Objectives

- Create sufficiently detailed mathematical models of wind turbine structures and lateral loading
  - Wind loading
  - Seismic loading
- Use the models to create benchmarks of load effects
  - Bending moments and shears in blades
  - Base shear and moment in tower
- Perform parametric studies on issues of interest
  - Seismic response
  - Fatigue characteristics

Wind Turbine Structures and Loading

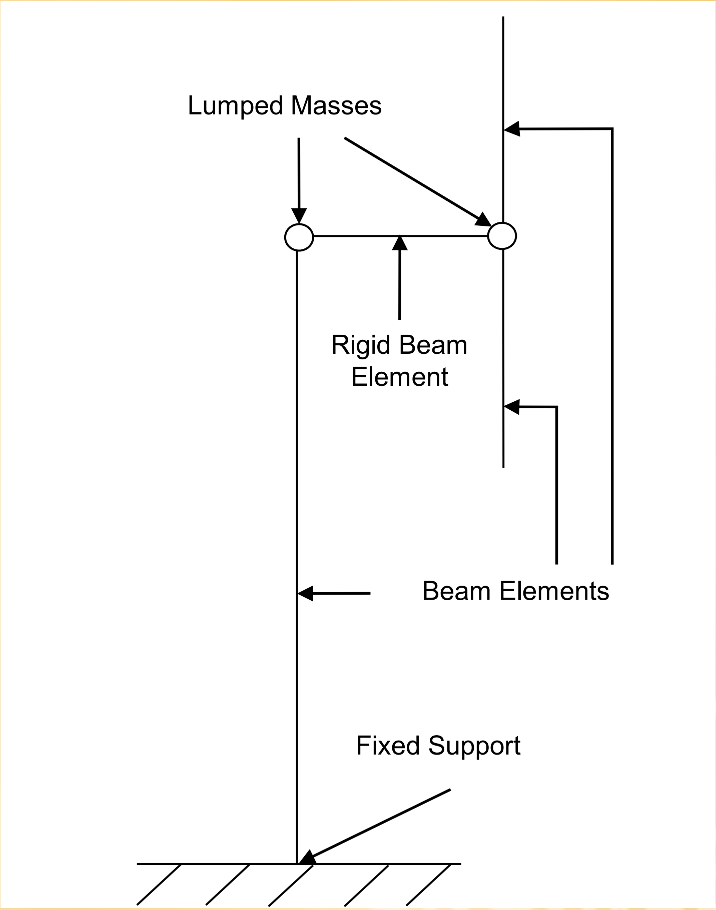
Wind Turbine Structures

- The tower is typically a tapered cylindrical steel shell
- The blades are typically constructed from glass-fibre reinforced plastic



Wind Turbine Structures (2)

- The tower and blades are long compared to their cross-sections, and can be considered as Euler beams
- All connections in this simple structural model are considered fixed



Turbine Loading – Wind Loads

- Wind loading is a broadband stochastic process which is typically described by a Power Spectral Density (PSD) function and coherence
- International Electrotechnical Commission (IEC) 61400-1, "Wind turbines. Part 1: Design requirements" describes the Kaimal (1972) spectrum for simulating wind speed time histories

$$PSD(f) = \frac{4\sigma^2 \frac{L}{V_{hub}}}{\left(1+6\frac{fL}{V_{hub}}\right)^{\frac{5}{3}}}$$

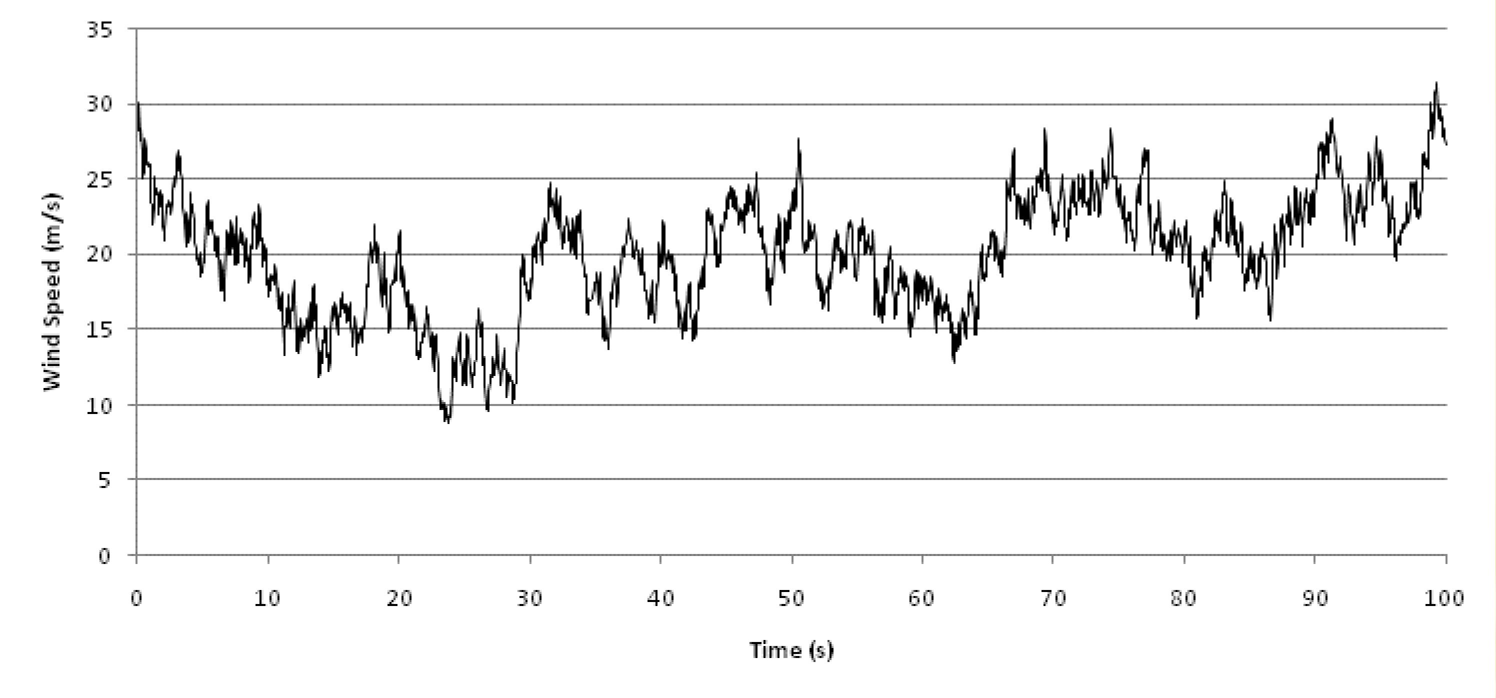
$$Coh(r,f) = exp\left[-12\left(\left(\frac{fr}{V_{hub}}\right)^2 + \left(\frac{0.12r}{L}\right)^2\right)^{0.5}\right]$$

f = frequency (Hz)
σ = standard deviation (m/s)
L = length scale (m)

r = distance between points (m)
V_{hub} = wind speed at hub height (m/s)

Turbine Loading – Wind Loads (2)

- A typical simulated wind speed time history is shown below:



Turbine Loading – Seismic Loads

- IEC 61400-1 does not provide minimum seismic requirements for standard turbines since seismic loads only govern in a few parts of the world
- Wind farms are being planned in seismically active areas such as California, the West Coast of Canada, and India, which warrant consideration of seismic loading

Turbine Loading – Seismic Loads (2)

- Seismic response can be determined using:
  - Response spectrum methods
  - Time-history analysis using measured ground accelerations
  - Time-history analysis using synthetic ground accelerations determined from PSD functions
- Time-history analysis is often used for structures in areas with significant seismic history

Structural Dynamics Formulation

Structural Dynamics Formulation - Governing Equations

- The governing equation for displacement of a continuous Euler beam is

$$m(x)\frac{\partial^2 u}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 u}{\partial x^2}\right]=p(x,t)$$

m = mass per unit length
u = displacement
t = time
x = location along beam
EI = flexural stiffness

- Displacement can be assumed to be equal to

$$u(x,t)=\sum_{i=1}^{\infty}\varphi_i(x)q_i(t)$$

where φ are known as the mode shapes and q are modal coordinates

Structural Dynamics Formulation - Governing Equations (2)

- The mode shapes are found by solving

$$[EI(x)\varphi''(x)]'' - \omega^2 m(x)\varphi(x) = 0$$

  where ω is the circular natural frequency (rad/s) for a given mode shape
- The modal coordinates are found by solving

$$\boldsymbol{M_G}\ddot{q} + \boldsymbol{C_G}\dot{q} + \boldsymbol{K_G}q = \boldsymbol{F_G}$$

$$M_{G\,i,j} = \int_0^L \varphi_i(x)m(x)\,\varphi_j(x)dx$$

$$K_{G\,i,j} = \int_0^L \varphi_i''(x)EI(x)\,\varphi_j''(x)dx$$

$$C_{G_{i,i}} = 2\zeta_i\omega_{ni}$$

$$F_{G\,i,1} = \int_0^L \varphi_i(x)p(x,t)dx$$

Structural Dynamics Formulation - Finite Element Formulation

- The structure is discretized into Euler beam finite elements which use cubic polynomial shape functions and matrices **M** and **K** are found
- Solving the eigenvalue problem

$$K - \omega_n^2 M = 0$$

  yields the mode shapes and natural frequencies
- The matrices required to find the modal coordinates are then

$$M_G = \varphi^T M \varphi$$
$$K_G = \varphi^T K \varphi$$

$$C_G = \varphi^T C \varphi$$

$$F_G = \varphi^T P$$

Matrix **C** is a linear combination of **M** and **K** (Rayleigh Damping)

Structural Dynamics Formulation - Wind and Seismic Loading

- Wind loads are determined using the basic expression

$$F_W = \frac{\rho C_D U^2 A}{2}$$

F_W = wind force
ρ = air density
C_D = drag coefficient
U = wind speed
A = cross-sectional area

- Seismic loading is applied to a structure in the form of acceleration at the base
- Seismic loading is applied as an effective inertial load

$$p_{eff}(x, y, z, t) = -m(x, y, z)a_g$$

where a_g is ground acceleration as a function of time

Structural Dynamics Formulation - Approximate Solutions

- If the natural frequencies of the blades and tower are well-separated, they can be idealized as separate structural systems:
  - The blades are assumed to be fixed-base cantilevered beams
  - The tower is assumed to be a fixed-base cantilevered beam carrying a point mass and inertia at its tip, representing the blade-hub-nacelle assembly

Use of COMSOL Multiphysics

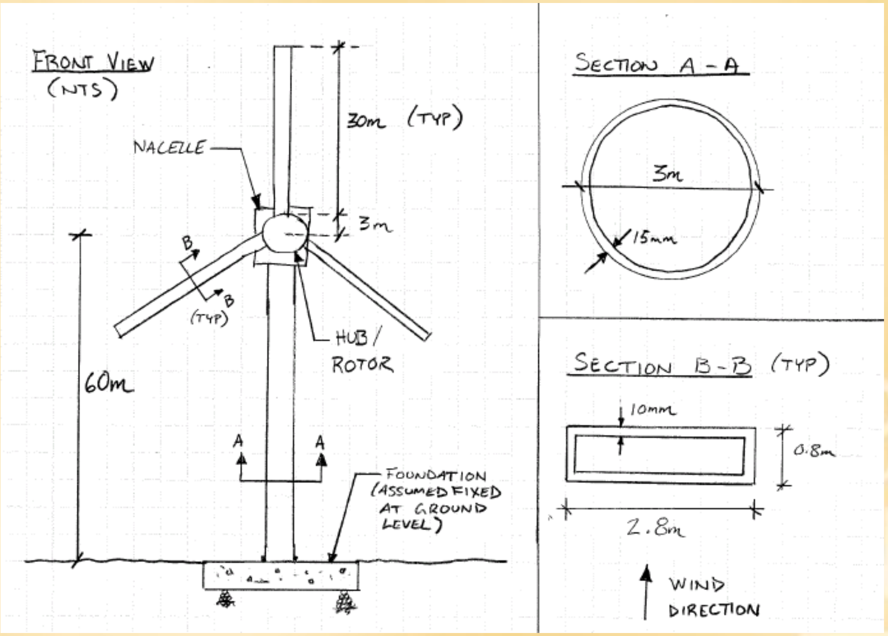
Use of COMSOL Multiphysics

- The turbine is idealized using the 3-D Euler Beam element in COMSOL, in the Structural Mechanics Module
- Definition of varied cross-sections such as tapered tower sections is accomplished easily
- Wind loads were simulated in MATLAB and input as functions into COMSOL
- Seismic ground acceleration was retrieved from Pacific Earthquake Engineering Research (PEER) Centre records and input as a function
- COMSOL offers direct solution of the equations of motion in addition to the modal superposition method described earlier

Turbine Simulations

Turbine Simulations

- A sample turbine model was subjected to wind and seismic loading, based on that discussed by Murtagh et al (2005)

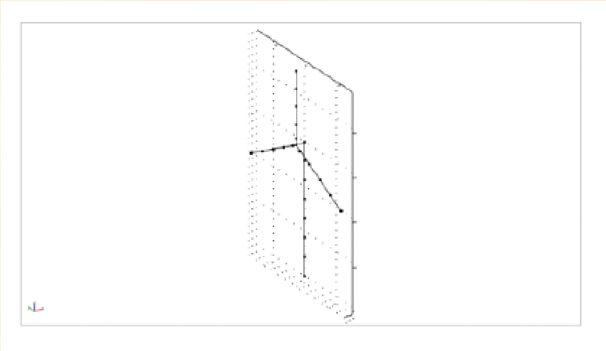


Turbine Simulations (2)

- Tower:
  - E = 210 Gpa
  - ρ = 7850 kg/m^3
  - C_D = 1.2
- Blades:
  - E = 650 Gpa
  - ρ = 2100 kg/m^3
  - C_D = 2.0
- Head offset = 4 m
- Hub-nacelle mass assumed to be 20,000 kg
- 2/3 of hub-nacelle assembly mass lumped at top of tower; 1/3 lumped at base of blades
- Mean wind speed at hub height is 20 m/s with a standard deviation of 2 m/s

Turbine Simulations (3)

- The model is considered stationary with one blade vertical as shown:



- Three wind time histories were simulated along each blade
- One component of the 1992 Cape Mendocino earthquake (Station CDMG, Petrolia) was applied which had a maximum acceleration of 0.41g

Results and Discussion

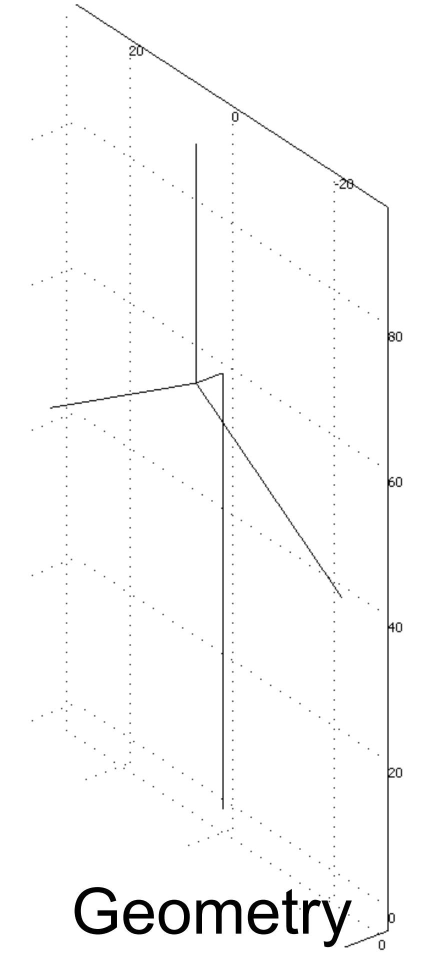
Results and Discussion – Natural Vibration Modes

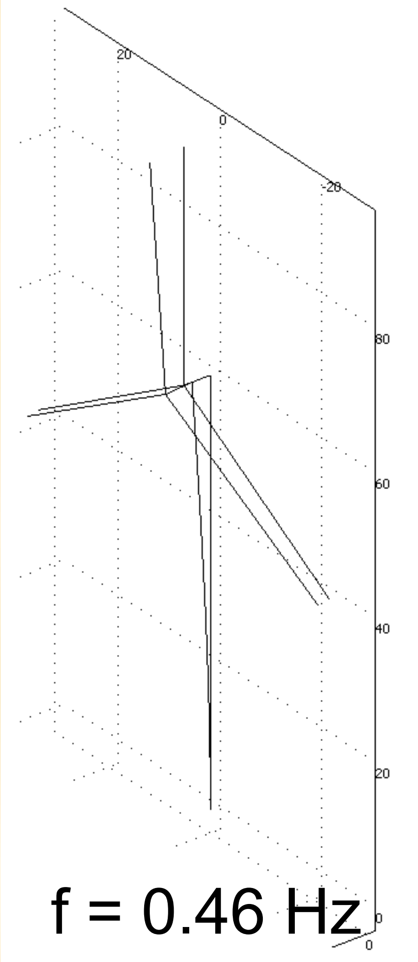
- COMSOL's Eigenfrequency solver was used to determine the modes and natural frequencies of the structure:

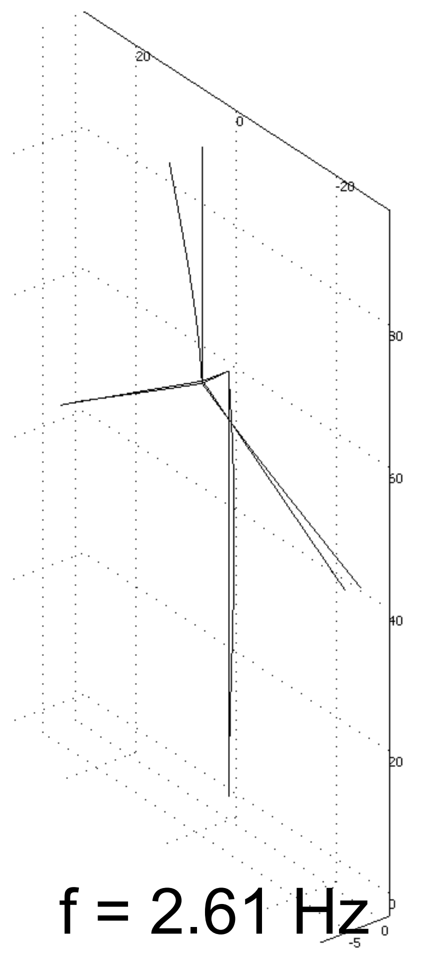
Frequency (Hz)	Description of Mode Shape
0.457	Tower bending across-wind
0.464	Tower bending along-wind
1.685	Tower torsion
2.495	Tower bending across-wind
2.608	Tower bending along-wind
4.204	Blade and tower bending
5.322	Blade and tower bending

- The first few frequencies correspond well to approximate solutions while the higher frequencies differ, indicating blade-tower coupling
- Rayleigh damping was implemented such that damping would be 1% of critical at 0.46 Hz and 5.0 Hz.

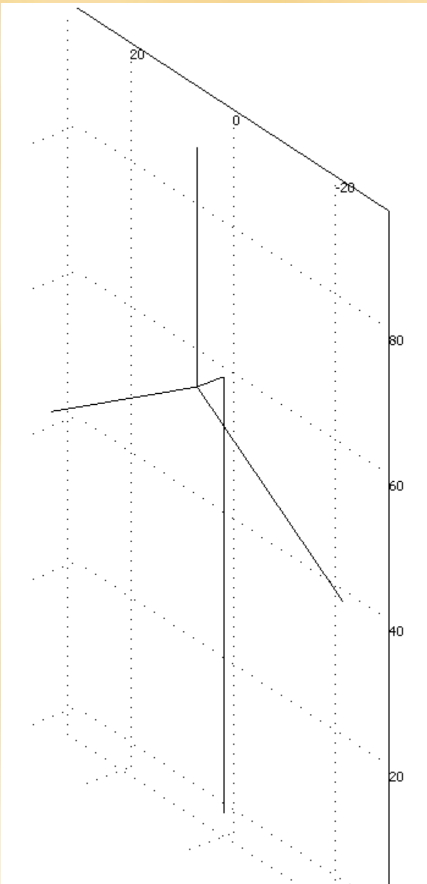
Results and Discussion – Natural Vibration Modes (2)



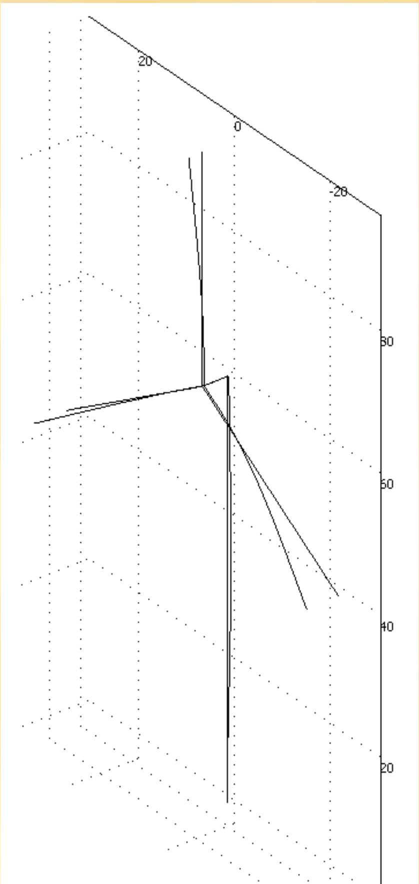




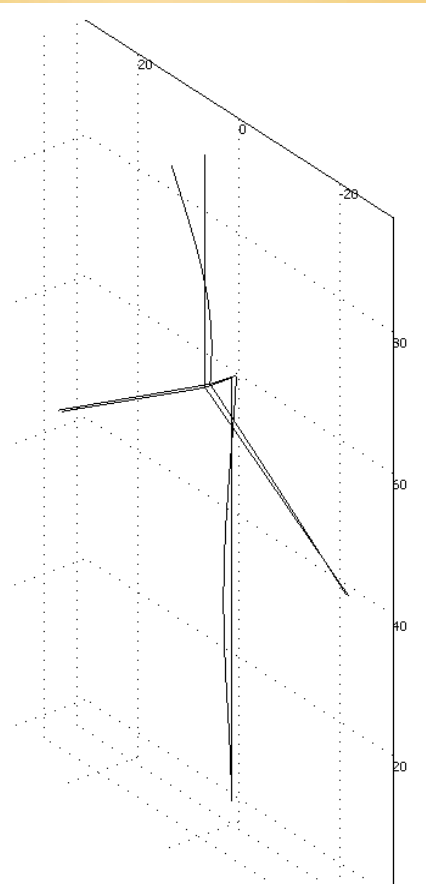
Results and Discussion – Natural Vibration Modes (3)



Geometry



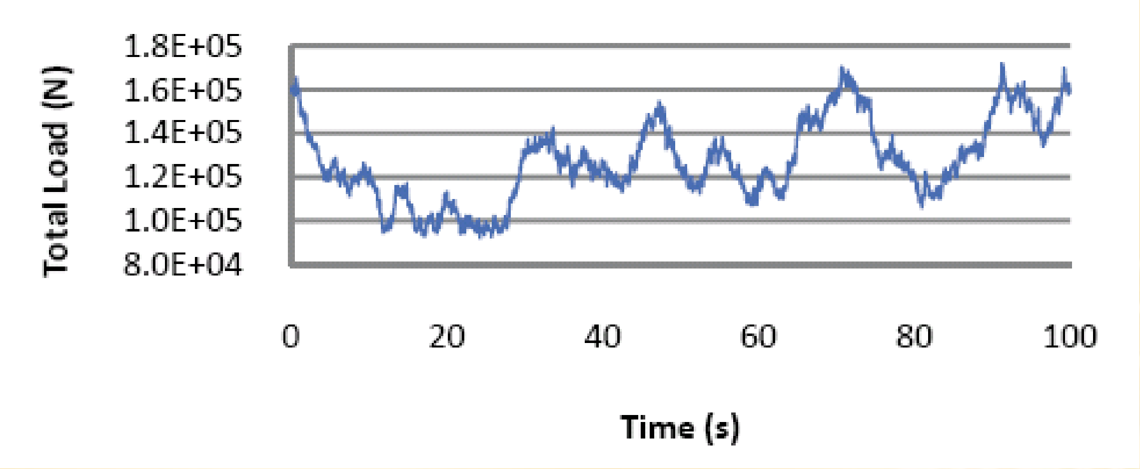
f = 4.20 Hz



f = 5.32 Hz

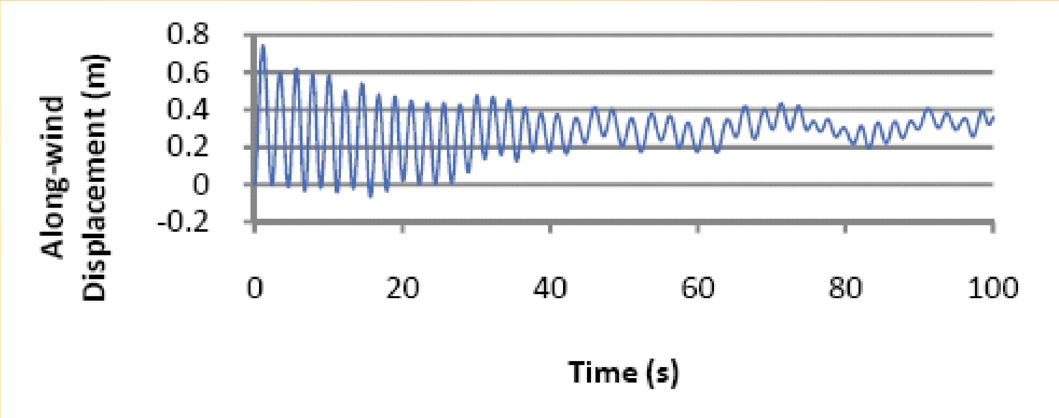
Results and Discussion – Response to Wind Loading

- The simulated wind loads on the blades were applied as uniform fluctuating loads over several blade sections
- The total load applied to the blades is shown below



Results and Discussion – Response to Wind Loading (2)

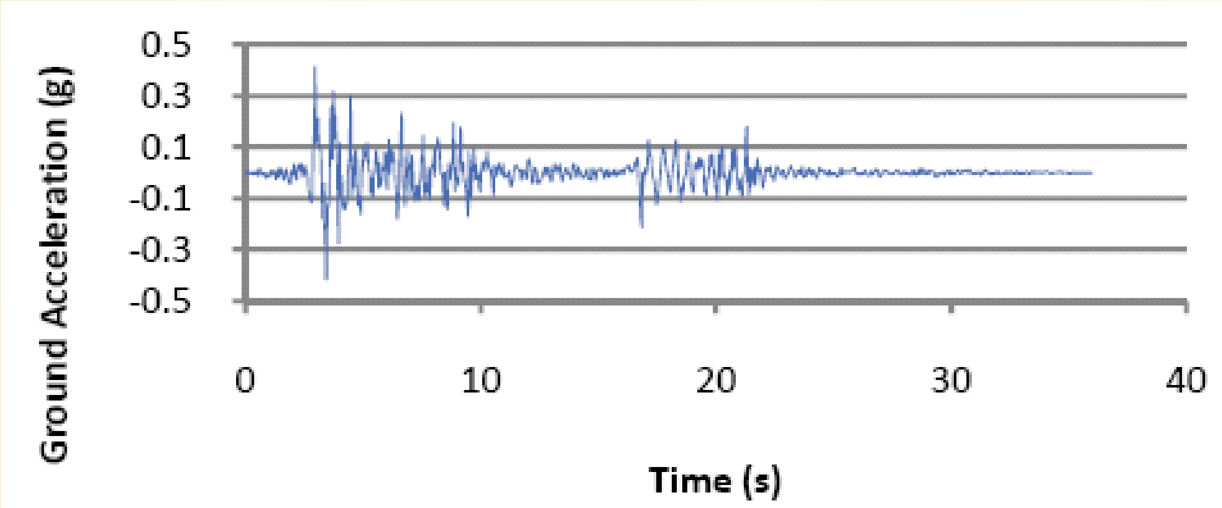
- The along-wind displacement at hub height is shown below



- The response after 40 s is approximately steady-state and varies mostly at the first tower bending frequency (0.46 Hz)
- The bending moment at the base of the tower had a similar time-history response

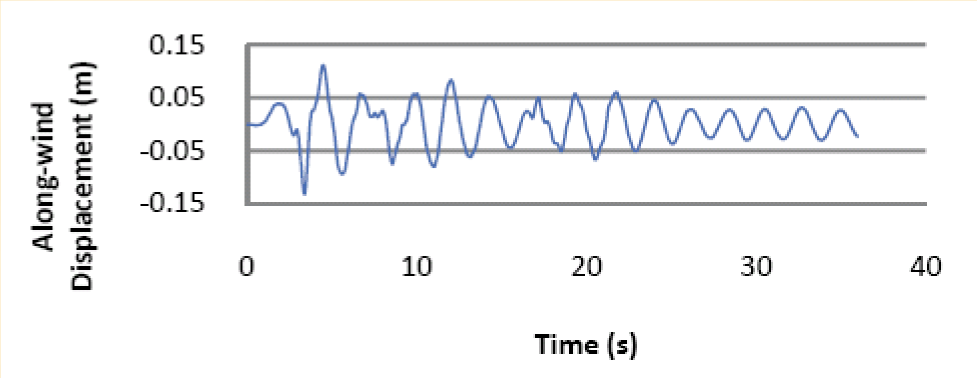
Results and Discussion – Response to Seismic Loading

- The recorded Cape Mendocino ground acceleration is shown below



Results and Discussion – Response to Seismic Loading (2)

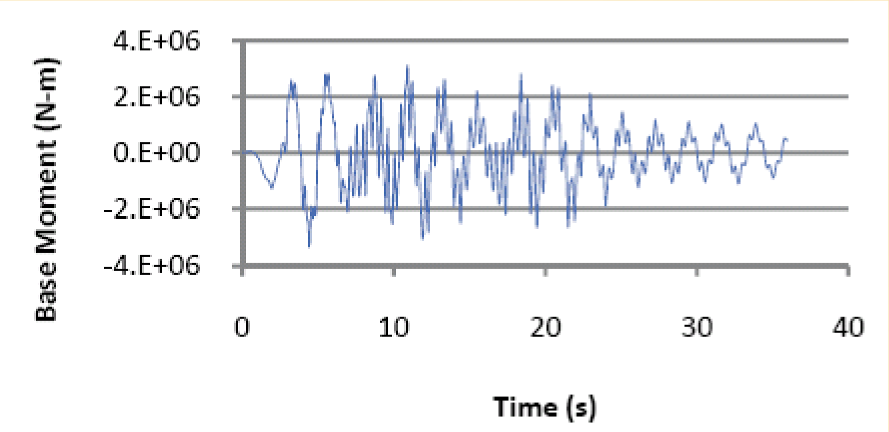
- The along-wind displacement at hub height is shown below



- The displacement varies mostly at the first tower bending frequency (0.46 Hz) but has some higher-frequency components

Results and Discussion – Response to Seismic Loading (3)

- The bending moment at the base of the tower is shown below



- The bending moment time-history displays more content at frequencies higher than the first natural tower bending mode

Conclusions and Future Work Using COMSOL

Conclusions

- This paper summarized the structure and loading on a wind turbine, presented a simple model and provided some simulation results
- Analysis of the sample turbine indicated that even if wind loads govern in magnitude over seismic loads, the response to seismic loading has significantly different frequency content than the response to wind loading, and warrants further detailed study

Future Work Using COMSOL

- Future research will use COMSOL for such aspects as:
  - Non-uniform turbine blades and tapered tower sections
  - Blade loading based on both lift and drag characteristics
  - Effects of blade rotation
  - Performance of various solutions to seismic loading and fatigue issues
  - Use of the MATLAB interface to perform intensive parametric studies

Acknowledgements

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Thank you!

Questions?