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Uncertainty Analysis, Verification and Validation of Stress Concentration in a Cantilever Beam

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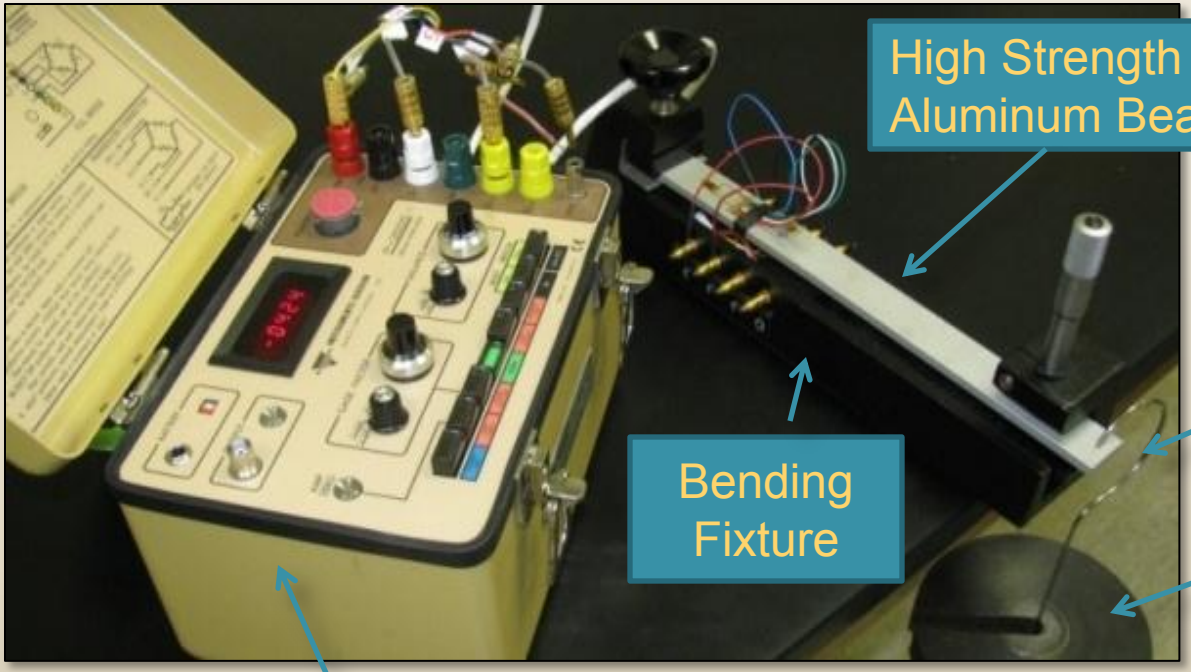
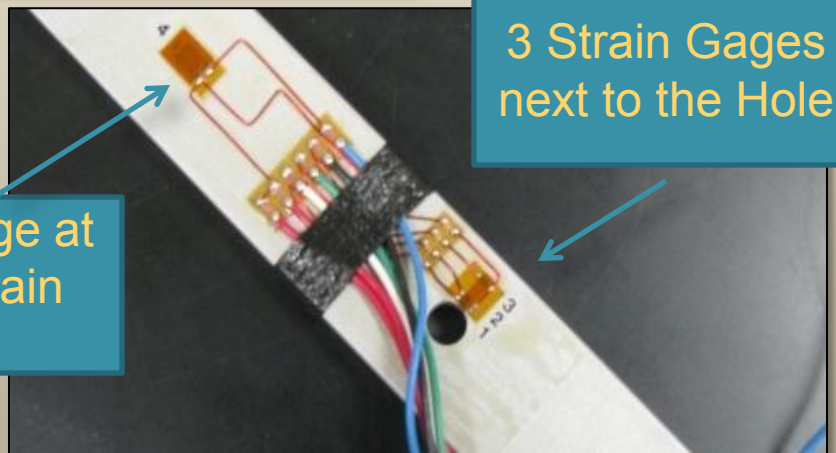
Introduction

- Stress Concentration in a Loaded Cantilever Beam which has a Hole
- Structural Mechanics Module in COMSOL Version 4.0a
- Simulations are Compared with Theoretical Values and Experimental Results
- Uncertainty in Simulation and Experimentation
- Monte Carlo Method was Used to Estimate the Total Uncertainty (Random & Systematic)
- Parametric Sweep Function in COMSOL

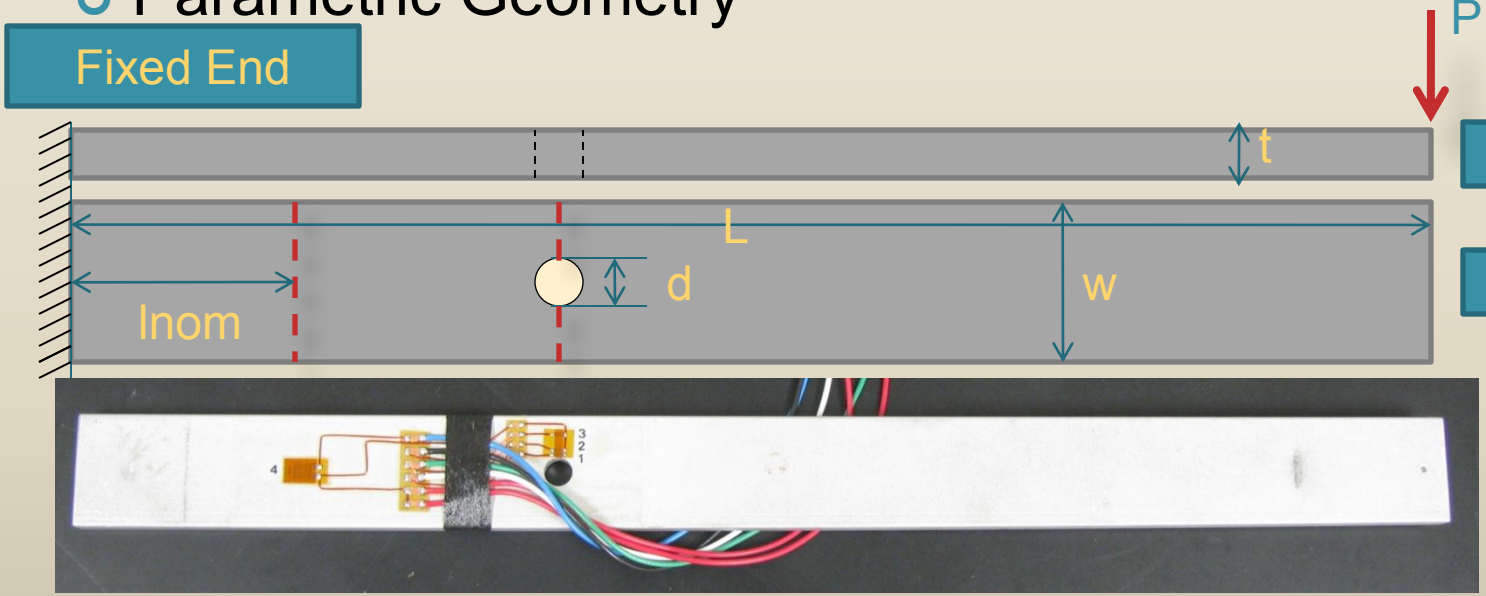
Experiment Setup

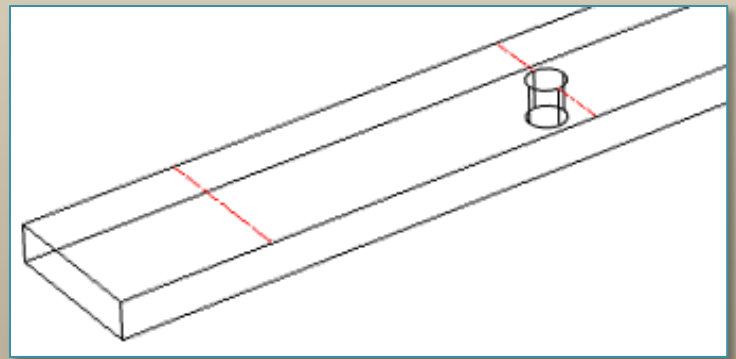




Simulation in COMSOL

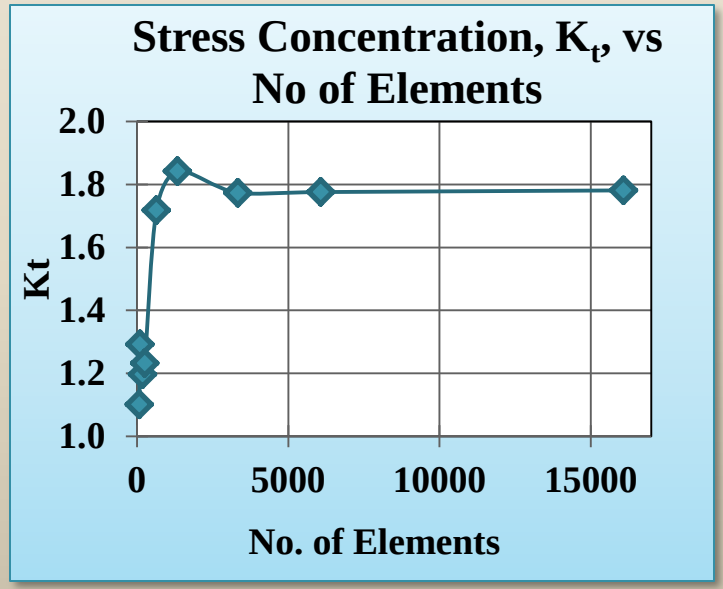
- CAD Device in 3-D Space was Used
- Parametric Geometry





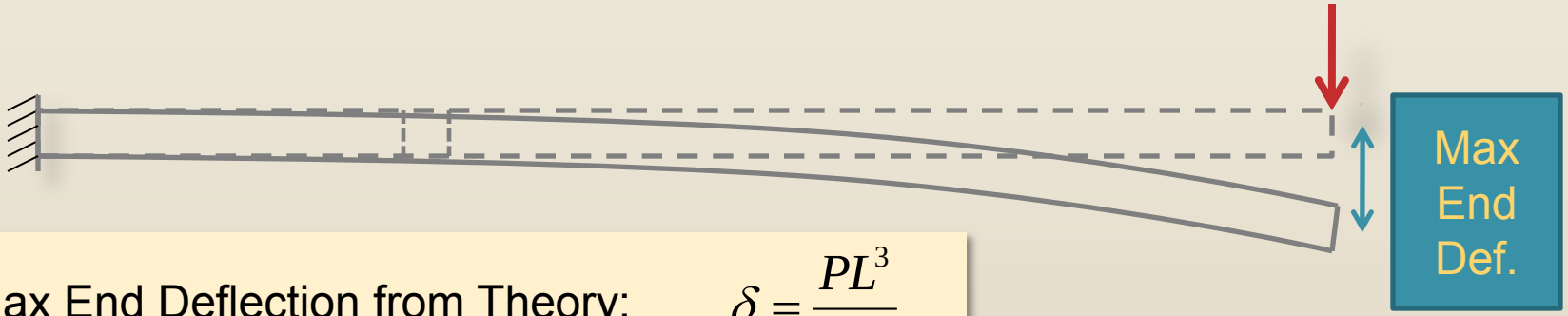
GRID CONVERGENCE STUDY

- Grid Convergence Study for Optimizing Tetrahedral Mesh Size for the Simulations.
- Nine Different Mesh Sizes



Mesh Size	Kt	No of Elements	Average Element Quality	Solution Time (s)
Extremely Coarse	1.100	84	0.2674	1.765
Extra Coarse	1.292	102	0.3396	1.656
Coarser	1.197	191	0.5082	1.563
Coarse	1.232	259	0.5874	1.593
Normal	1.718	644	0.7070	1.734
Fine	1.842	1340	0.7377	1.984
Finer	1.773	3340	0.7746	2.500
Extra Fine	1.776	6082	0.7950	2.860
Extremely Fine	1.781	16080	0.8213	4.422

Simulation Verification

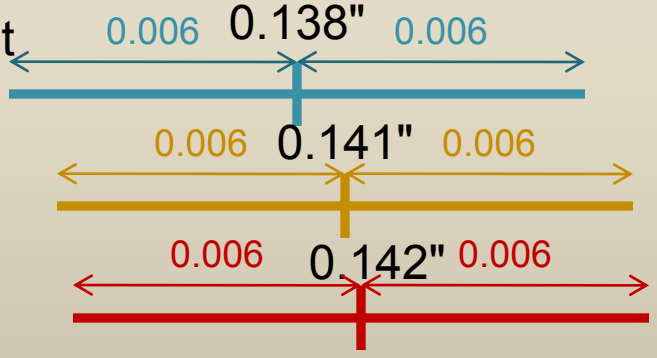


Max End Deflection from Theory: $\delta = \frac{PL^3}{3EI}$

Simulation, Beam without the Hole

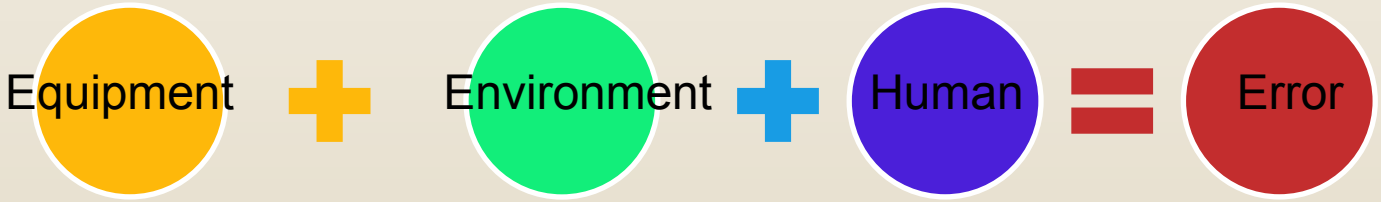
Theory

Simulation, Beam with the Hole



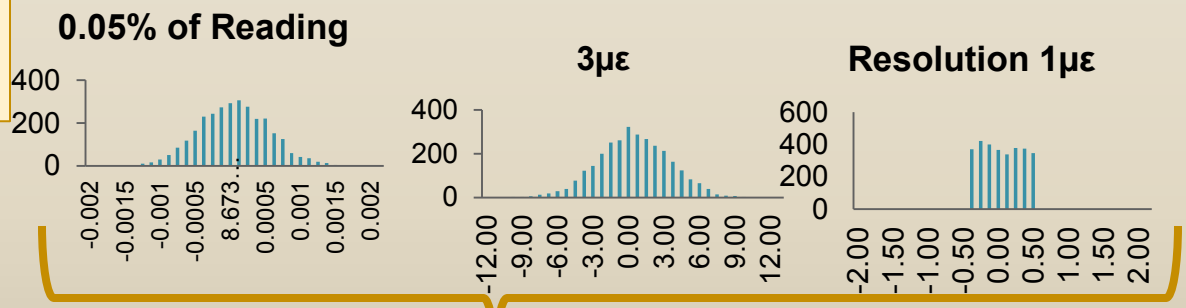
- Both Simulations Bracket the Theoretical Value of Deflection.

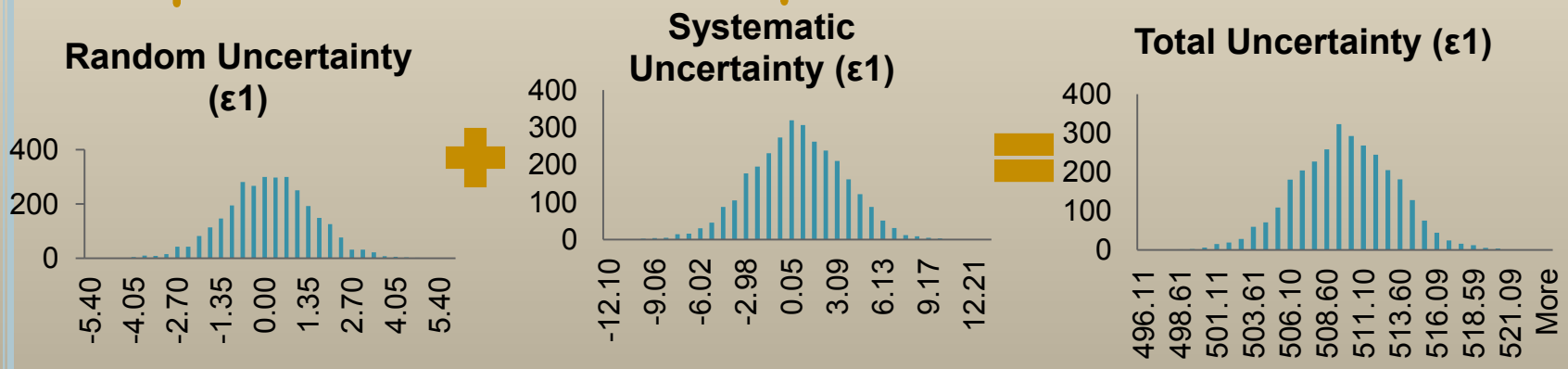
Uncertainty in Experiment



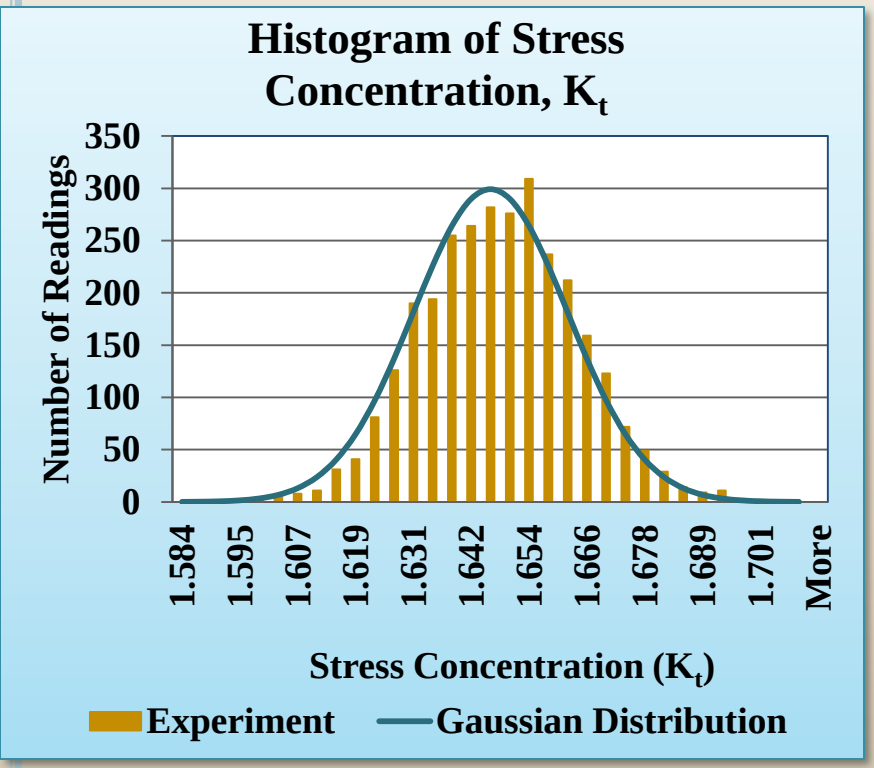
Repeating Experiment 10 times

Accuracy and Resolution of Equipment

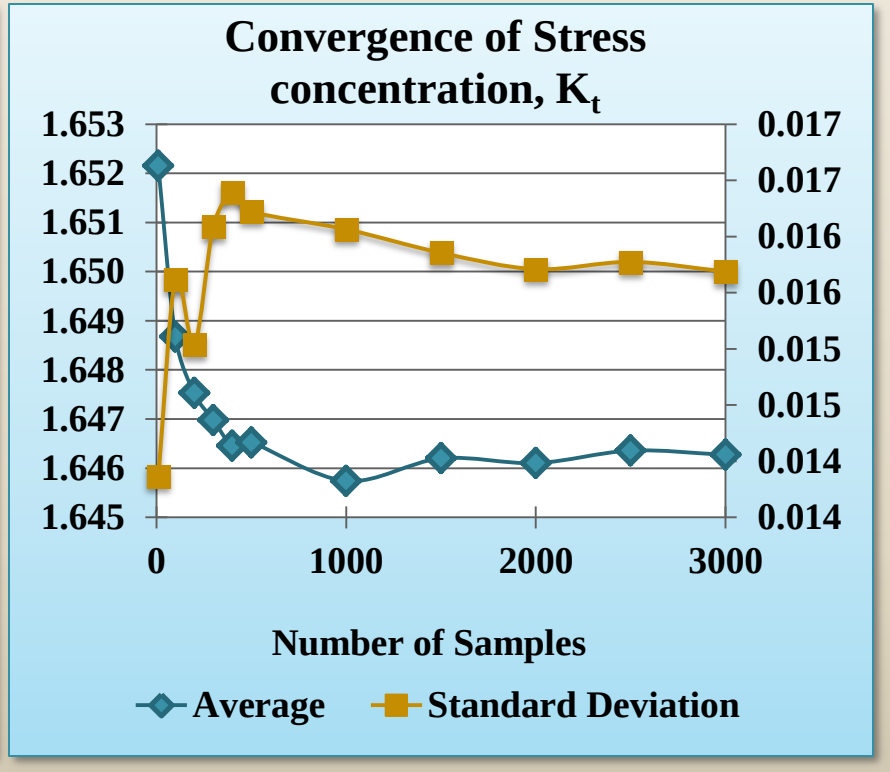




EXPERIMENTAL UNCERTAINTY ANALYSIS

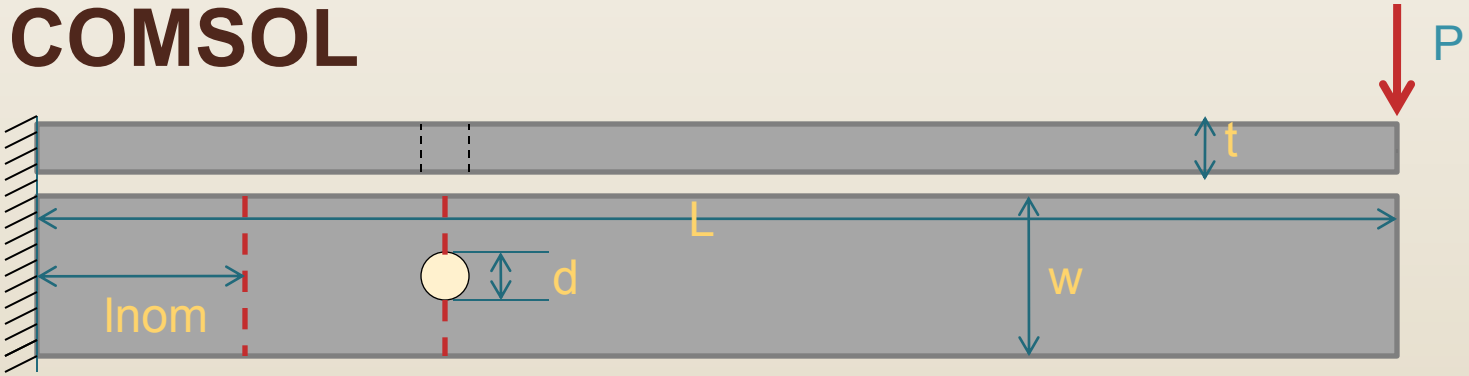


Histogram of K_t for the Experiment

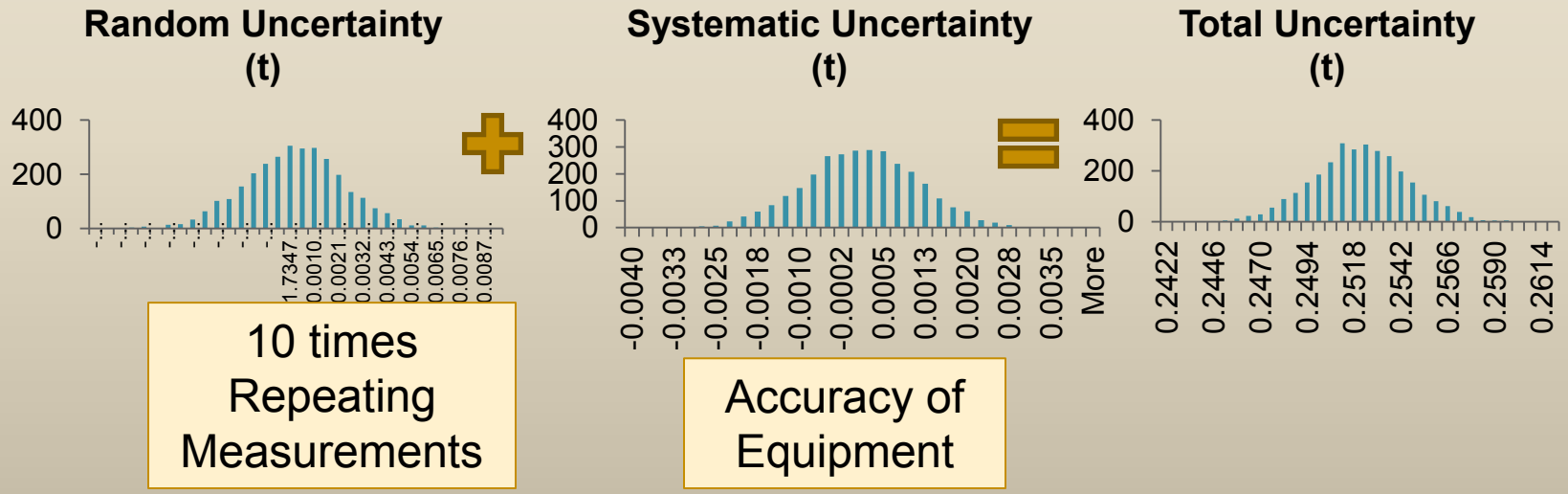


Convergence of Average and Standard Deviation of K_t for Experiment

Uncertainty in Simulation with COMSOL



- Input Parameters: t, L, w, d, lnom, P, E



10 times Repeating Measurements

Accuracy of Equipment

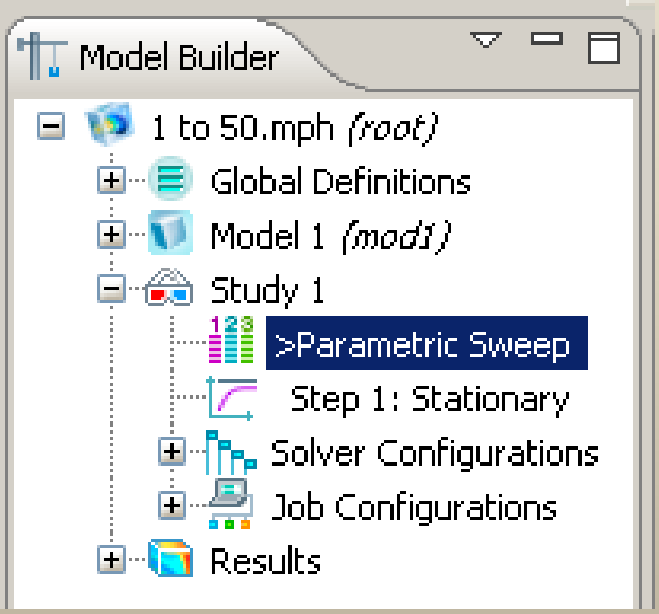
- Monte Carlo, 3000 Samples → Time Consuming

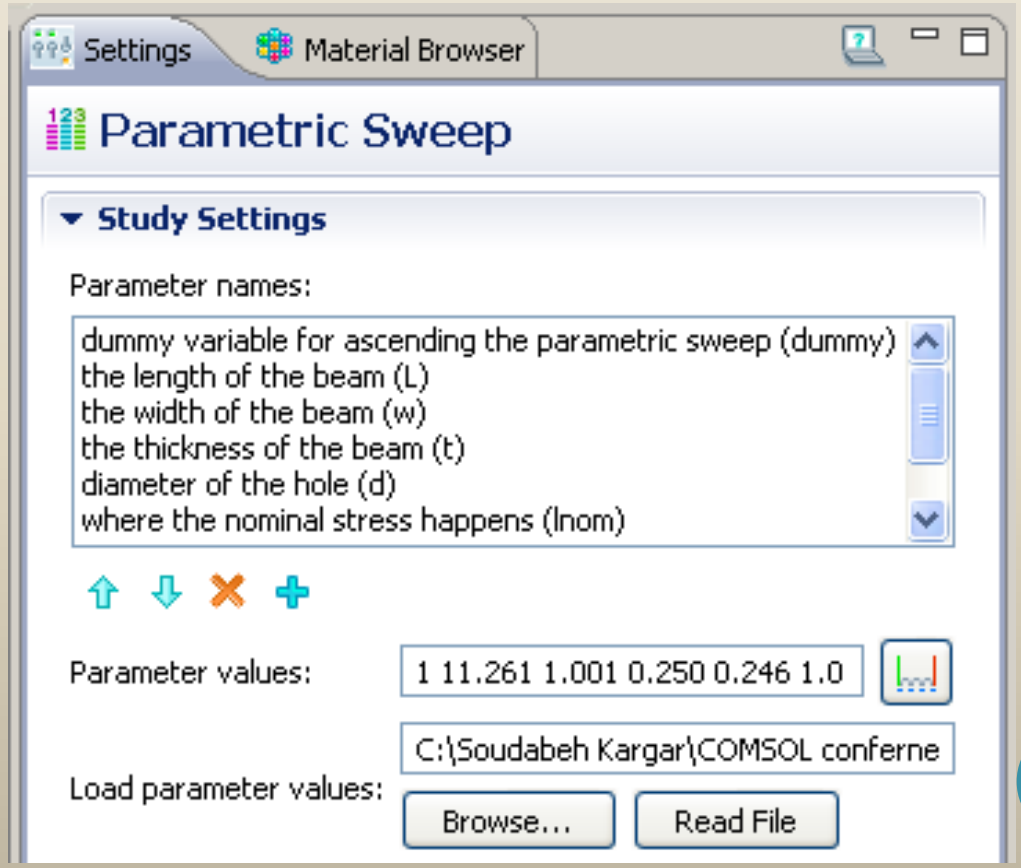
PARAMETRIC SWEEP

- Variables in Parametric Format

Optimizing
Uncertainty Studies

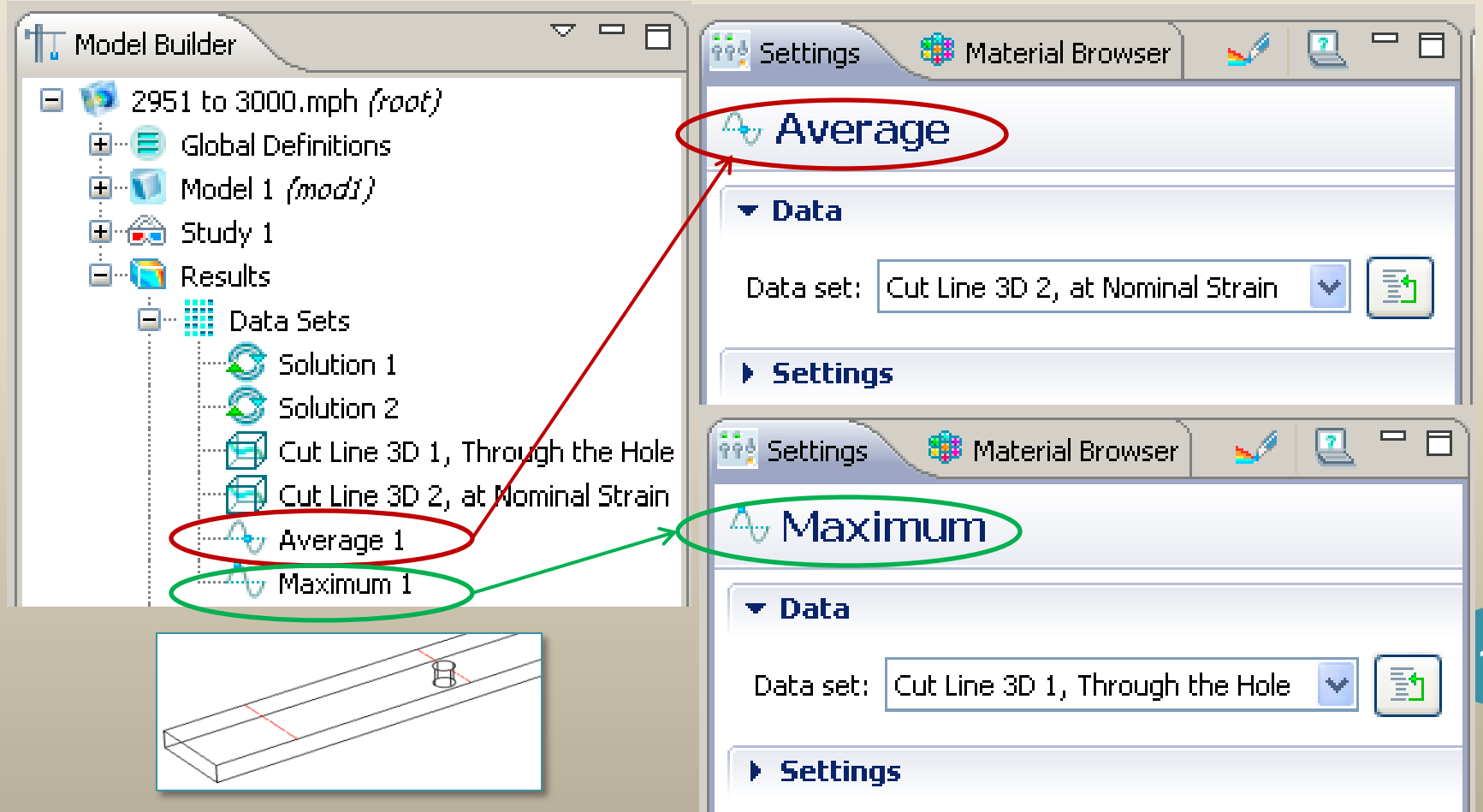
Geometry
Material Properties
Boundary Conditions



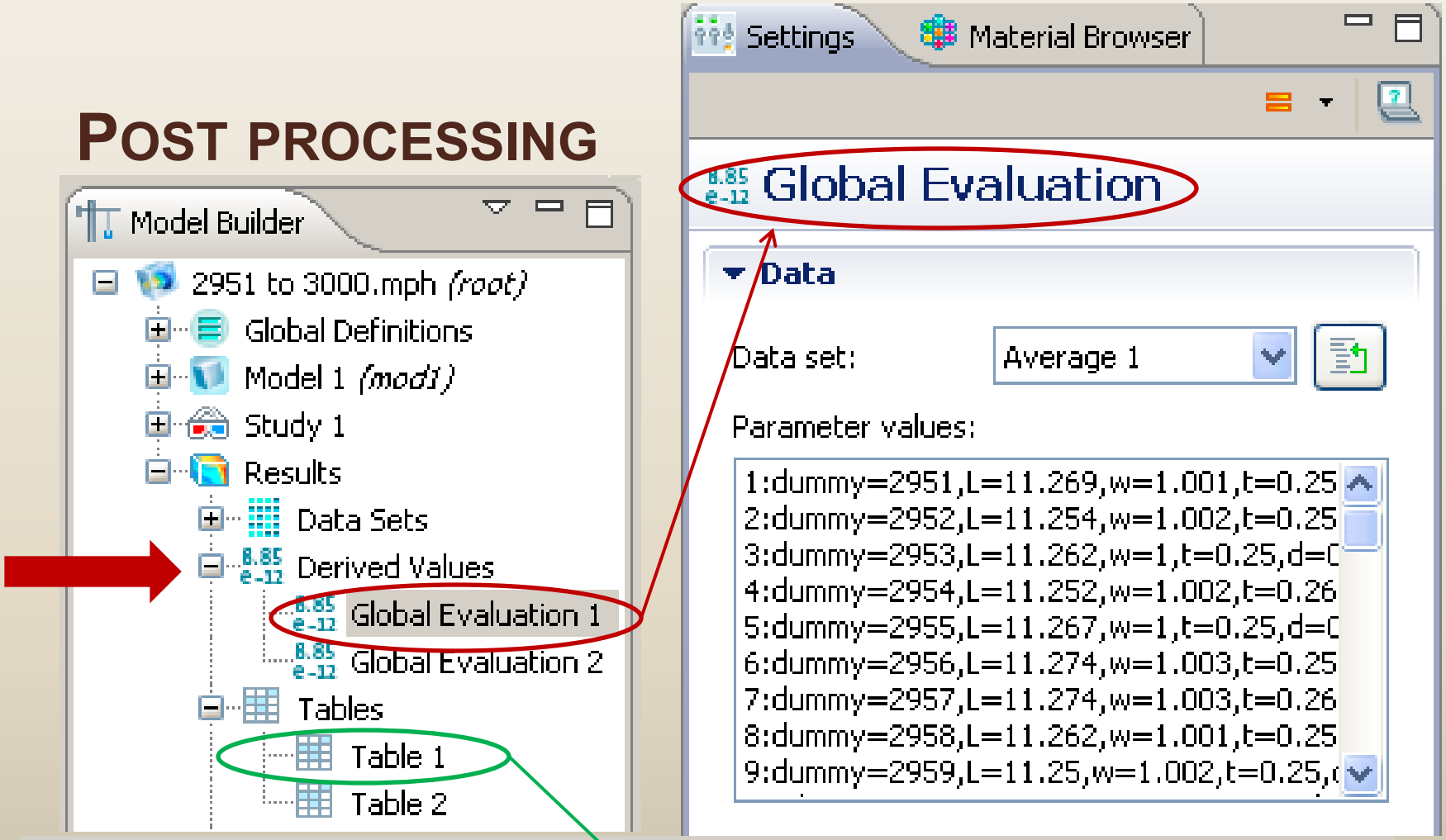


POST PROCESSING

- The Stress or Strain Concentration Factor: $K_t = \frac{\varepsilon_{max}}{\varepsilon_{nom}}$



POST PROCESSING

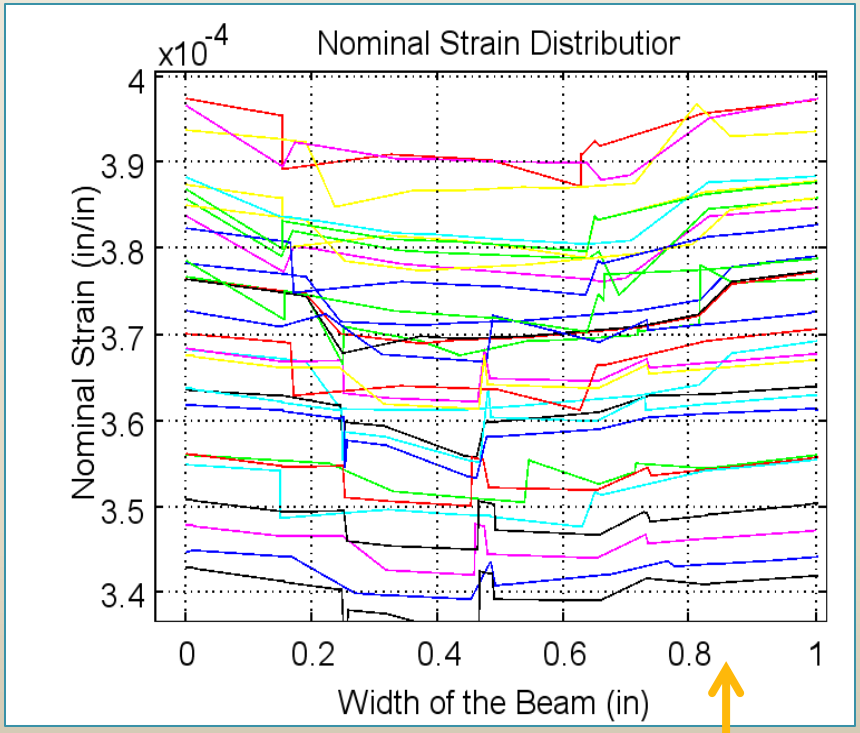


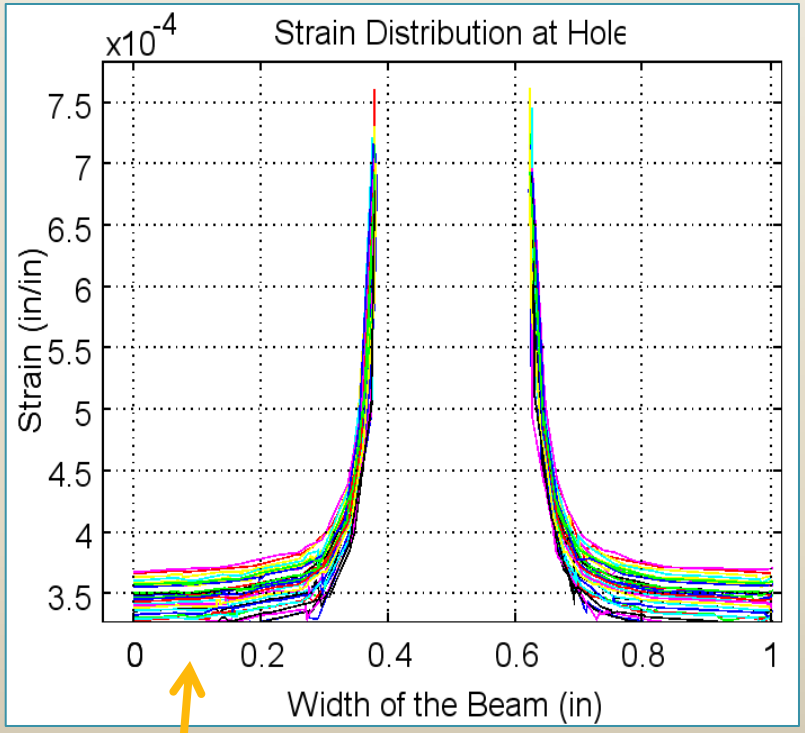
Messages | Progress | Results

dummy	L	w	t	d	Inom	P	Eud	Total displa
1	11.261	1.001	0.25	0.246	1.01	18.235	7.12e10	0.1455
2	11.268	1	0.25	0.246	1.02	18.221	6.94e10	0.1495
3	11.258	1.001	0.252	0.241	1.025	18.127	7.21e10	0.1393
4	11.257	1	0.249	0.243	1.018	18.104	7.4e10	0.1407
5	11.266	1.001	0.249	0.243	1.014	18.072	7.39e10	0.1407

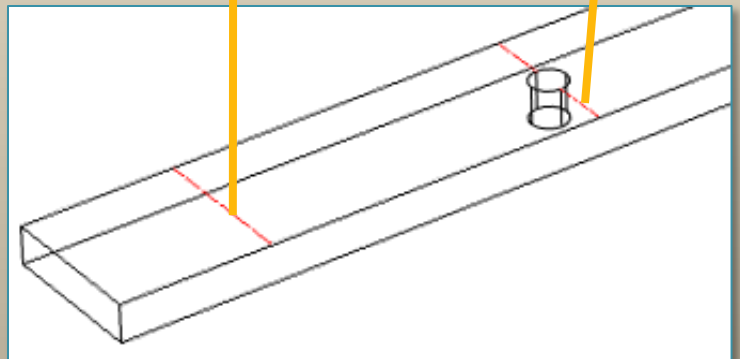
STRAIN DISTRIBUTION

$$K_t = \frac{\varepsilon_{max}}{\varepsilon_{nom}}$$





Strain Distribution at Nominal Strain



Strain Distribution at the Hole

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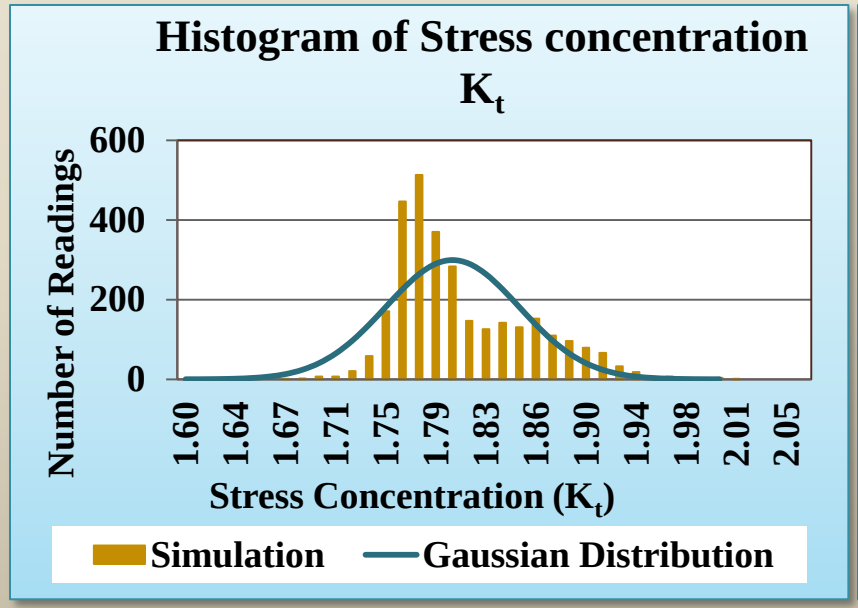
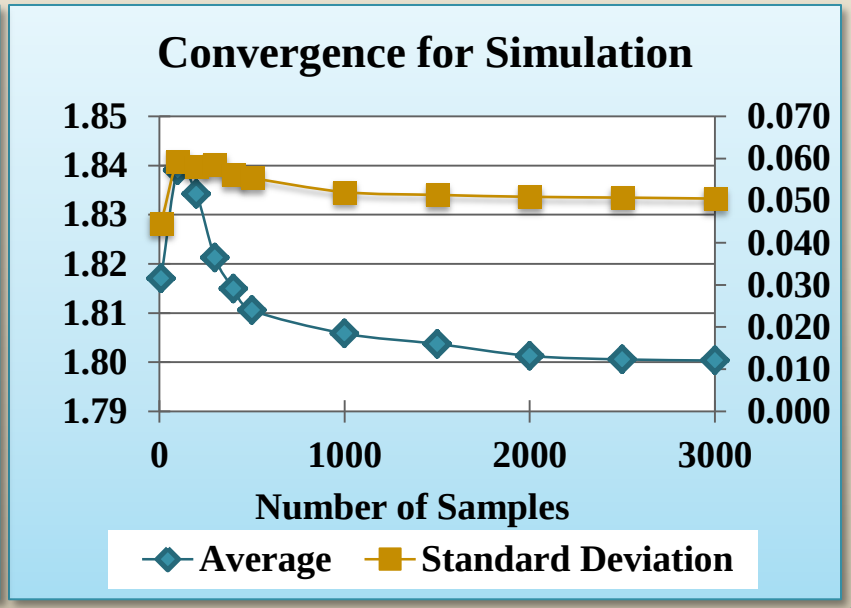
Convergence of number of samples in simulation

- Computing Power, a Controlling Factor
- Importance of Convergence Study
- Simple versus Complicated Models

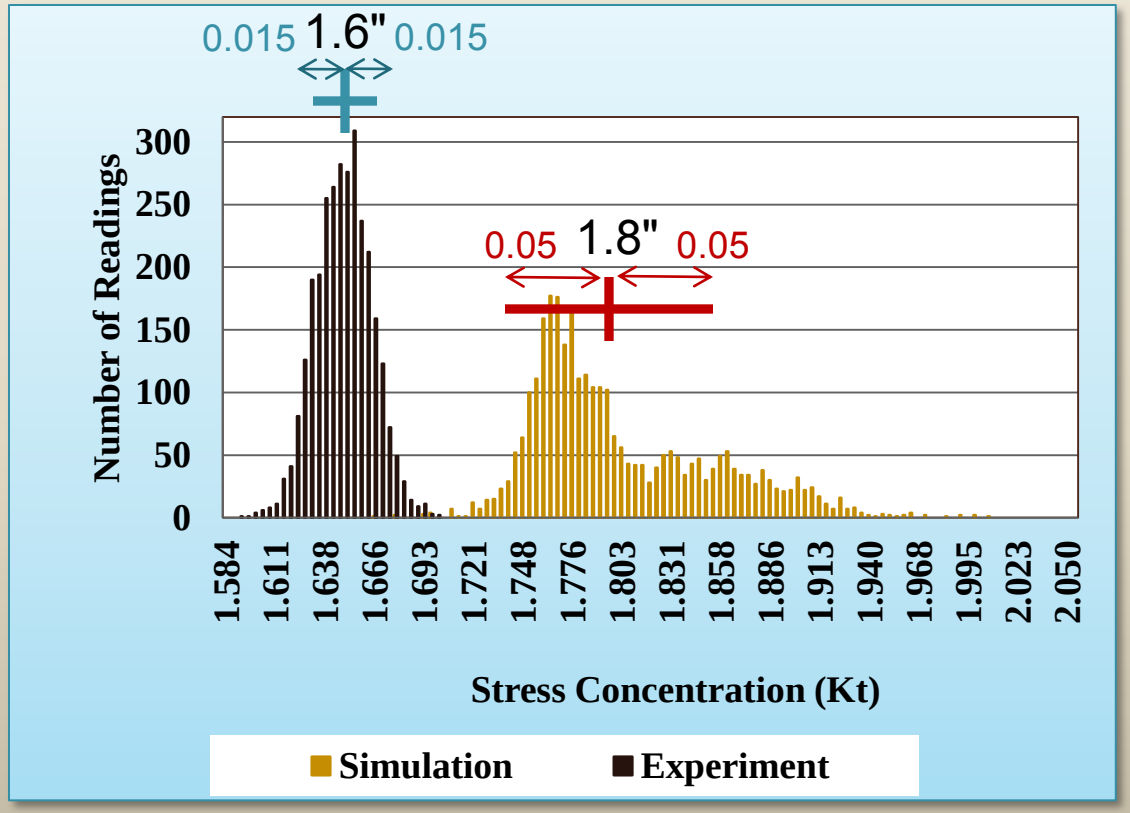


Histogram of K_t for Uncertainty in COMSOL Simulation in Comparison to Gaussian Distribution.



Convergence of Average and Standard Deviation of K_t for COMSOL Simulation.

Comparison of Histograms



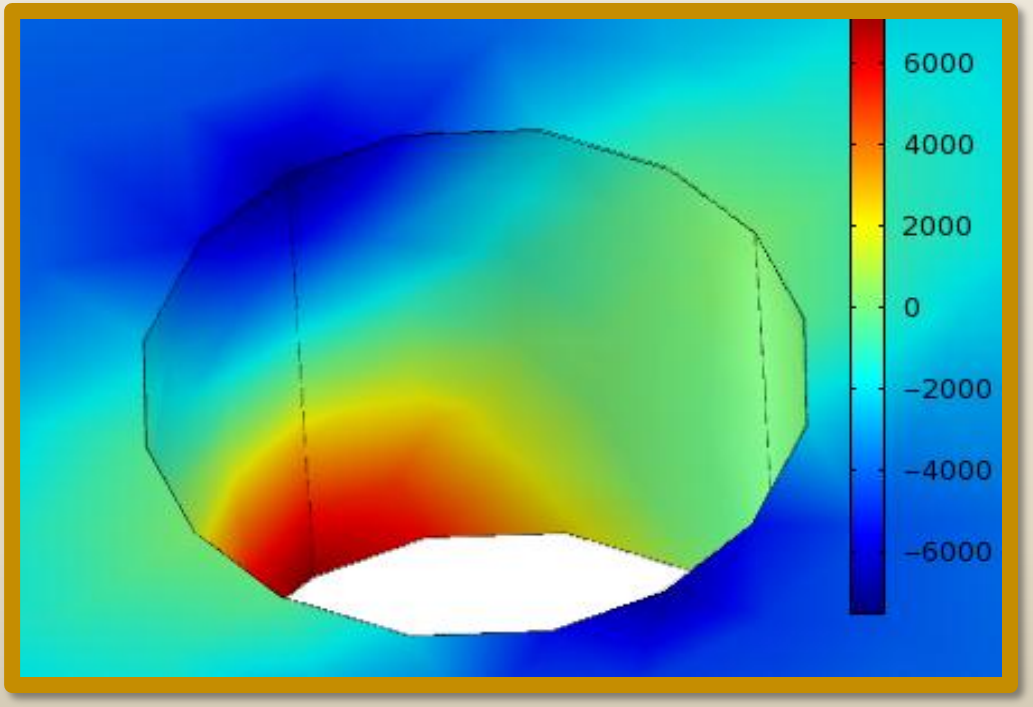
- Histogram of K_t for Experiment and Simulation

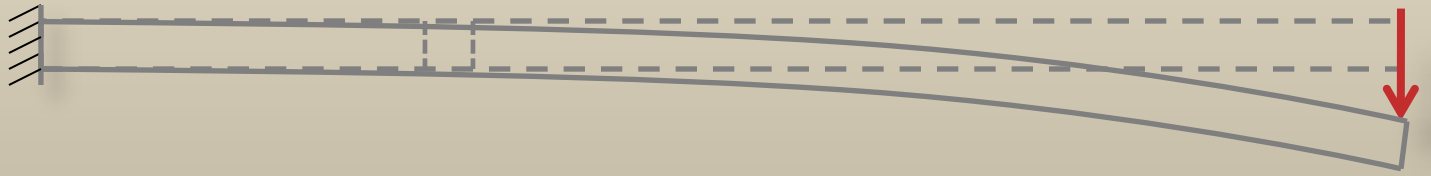
SUMMARY AND RESULTS

- In this Paper, we Looked at Experimentation, Computational Simulation and the Uncertainty Propagation.
- The Stress Concentration Factor:
  - In Experiment :1.646 ± 0.0141 and 1.646 ± 0.0157
  - In Simulation: 1.80 ± 0.050.
  - Thus we have a Comparison Error[6] Implying there is an Un-modeled, Un-simulated Effect such as the Strain Gauge Sensor Glue; or, Perhaps there is an Experimental or Input Uncertainty that is not Captured.
- Parametric Sweep in COMSOL is a Simple Tool to Perform Uncertainty Analysis with Monte Carlo Technique in our Computational Simulations.
- The Same Method can be Used to Learn which Parameter has More Effect on the Final Result.
- In Order to Prevent Consuming Time and Money on Doing Different Experiments with Different Uncertainty in Parameters, we can use Uncertainty Analysis in Simulation to Find out which Parameter(s) are Controlling Factors.
- Defining Cut-line in Parametric format causes Problem.

References

- 1. Hugh W. Coleman, W Glenn Steele, *Experimentation, Validation, and Uncertainty Analysis for Engineers 3rd Edition*, page 64, John Wiley & Sons, Inc., Hoboken, New Jersey (2009).
- 2. Instrumentation Division Measurements Group, INC, *P-3500 Digital Strain Indicator Instruction Manual*, page 8 and 14.
- 3. Introduction to COMSOL Multiphysics Version 4.0a (2010)
- 4. S. P. Timoshenko, J. N. Goodier, *Theory of Elasticity*, page 26, 27, 28, 65, 66, 67, 68, 90, 91, 92. McGraw-Hill Book Company, New York (1970).
- 5. J. A. Gilbert, C. L. Carmen, *MAE/CE 370 Mechanics of Materials Laboratory Manual Version 1.0*, page numbers. Department of Mechanical and Aerospace Engineering, University of Alabama in Huntsville (2000).
- 6.The American Society of Mechanical Engineers, ASME V&V 20-2009 – Standard for Verification and Validation in Computational Fluids and Heat Transfer", November 2009.





THANK YOU!

Stress Distribution around Circular Holes

- **Strain Distribution:**

$$\varepsilon_i = A + B\left[\frac{R}{Z_i}\right]^2 + C\left[\frac{R}{Z_i}\right]^4$$

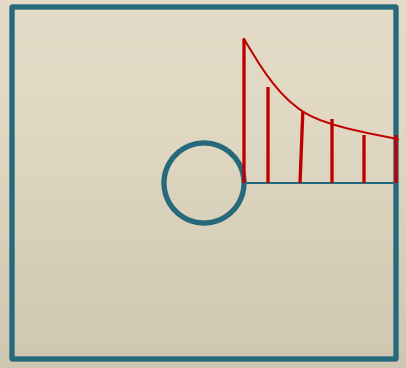
R(in)	Z1(in)	Z2(in)	Z3(in)
0.125	0.145	0.185	0.325

R: Radius of Hole
Zi: Distance from Strain Gage to the Center of the Hole
A,B,C: Constants

$$C = 5.86(\varepsilon_1 - \varepsilon_2) - 5.44(\varepsilon_2 - \varepsilon_3)$$

$$B = 3.49(\varepsilon_1 - \varepsilon_2) - 1.2C$$

$$A = \varepsilon_1 - 0.743B - 0.522C$$



- **At the Edge of the Hole where,**

$$\frac{R}{Z} = 1$$

$$\varepsilon_{\max} = A + B + C$$

Experimental Results

- The Uncertainty of K_t is Found by Two Methods:
  - 1) Random and Systematic Uncertainties for K_t are Calculated as Standard Deviation and then are Combined with:

$$u_{Kt} = \sqrt{b_{Kt}^{\ 2} + S_{Kt}^{\ 2}}$$

  - 2) Value of K_t is Found for All 3000 Samples Including the Random and Systematic Uncertainty and then the Total Uncertainty for K_t is Calculated by Calculating the Standard Deviation
- The Average Value of K_t for both Methods:
  - Average = 1.646
- The Total Uncertainty for both Methods:
  - Method (1) = 0.0141
  - Method (2) = 0.0157

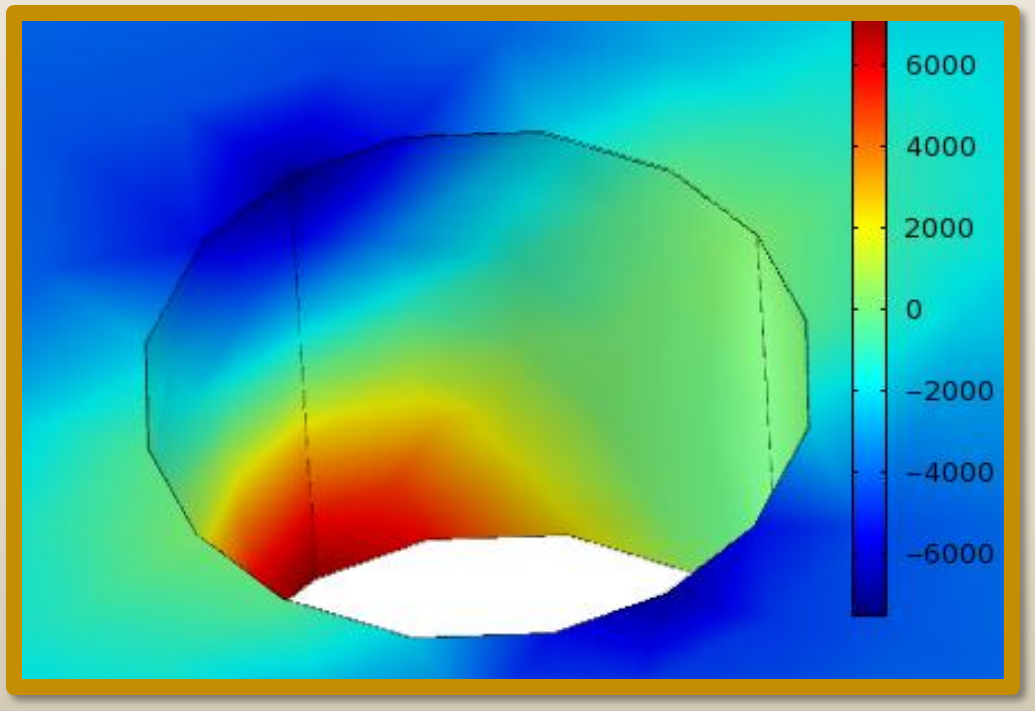
Parametric Sweep

- Defined Variables for:
  - Random Error
  - Systematic Error

dummy	Dummy Variable in Ascending Order
L	Length of the Beam
w	Width of the Beam
t	Thickness of the Beam
d	Diameter of the Hole
lnom	Distance, Nominal Stress to Fixed End
P	Load
E	Modulus of Elasticity

Parameter	Average	Random Error	Systematic Error
L	11.260	0.009	0.003
w	1.002	0.001	0.001
t	0.252	0.002	0.001
d	0.246	0.002	0.001
lnom	1.021	0.005	0.001
P1	4.010	0.000	0.010
P2	0.064	0.002	0.010
E	1.04E+07	2.5%	0

3 DIMENSIONAL GRAPH OF THE STRESS DISTRIBUTION AROUND THE HOLE



Strain Distribution at the Hole