Identification of the Complex Moduli of Orthotropic Materials using Modal Analysis

Filip Van den Abeele[*,1], Jose Rogerio De Oliveira Jr.[2] and Francisco J. Huertos[1]

[1]OCAS N.V., J.F. Kennedylaan 3, BE-9060 Zelzate, Belgium
[2]ArcelorMittal Inox Brasil S.A., Timóteo, Brasil
*Corresponding author: filip.vandenabeele@arcelormittal.com

Abstract: It is very difficult –if not impossible- to measure the global (homogenized) properties of heterogeneous and anisotropic materials like fibre reinforced plastics, composites or sandwich structures. When designing composite sandwich applications, the elastic properties are required to perform structural stiffness and strength calculations. However, due to the anisotropic nature of composite materials, it is not straightforward to measure these properties with traditional mechanical characterization.
In this paper, the Resonalyser method is introduced as a material identification technique following a reverse engineering scheme. It is a non destructive test methodology to measure the average elastic properties of anisotropic materials. In plane stress conditions, the elastic behaviour of an orthotropic material can be described by four engineering constants: the Young's moduli E_1 and E_2, the shear stiffness G_{12} and the contraction coefficient ν_{12}. These in-plane elastic properties can be determined by a dynamic modulus identification technique using resonant frequencies. Both the determination of the real part (elastic constants) and the imaginary part (damping behaviour) are based on the measurement of the vibrational response of a rectangular test plate, submitted to a controlled excitation.
In the Resonalyser method, the measured frequencies are compared with the computed resonance frequencies of a numerical parameter model of the test plate. The parameters in the model are the unknown elastic properties. Starting from an initial guess, the parameters of the numerical model are tuned until the computed resonance frequencies match the measured ones. This tuning technique is based on the sensitivities of the resonant frequencies for parameter changes. The obtained elastic material properties are homogenised over the plate surface, and hence suitable as averaged input values in finite element models to perform stiffness and strength calculations.

Keywords: non destructive testing, composite materials, orthotropic properties, modal analysis, reverse engineering, material identification

1. Introduction

Engineers are always pushing the envelope when trying to design lightweight structures with outstanding mechanical performance. Sandwich panels with high strength steel skins provide an innovative solution for these stringent design requirements. Such sandwich structures, schematically shown on Figure 1, combine the best of both worlds: high strength steel skins with a thermoplastic core. They can be used for transport applications, in the building industry, for storage and packaging, office furniture, visual communication boards,...

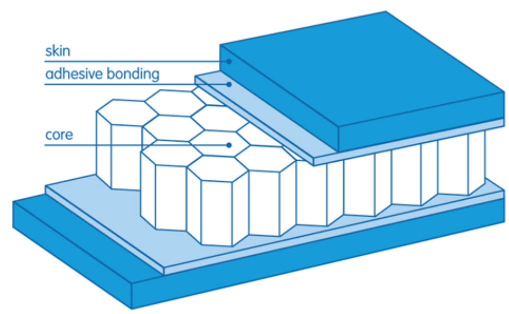

Figure 1. Elytra sandwich structures with steel skins.

When designing composite sandwich applications, the elastic properties are required to perform structural stiffness and strength calculations. However, due to its honeycomb core and layered composition, the material properties of sandwich structures are anisotropic or direction dependent. It is not straightforward (if not impossible) to obtain these properties with traditional mechanical characterization. The Resonalyser method [01] provides an elegant reverse engineering scheme to determine all elastic constants with only one non destructive test.

In this paper, the Resonalyser method is introduced as a material identification technique following a reverse engineering scheme. The in-plane elastic properties $\{E_1, E_2, G_{12}, \nu_{12}\}$ can be determined by a dynamic modulus identification using resonant frequencies [02]. Both the determination of the real part (elastic constants) and the imaginary part (damping behaviour) are based on the measurement of the vibrational response of a rectangular test plate, submitted to a controlled excitation.

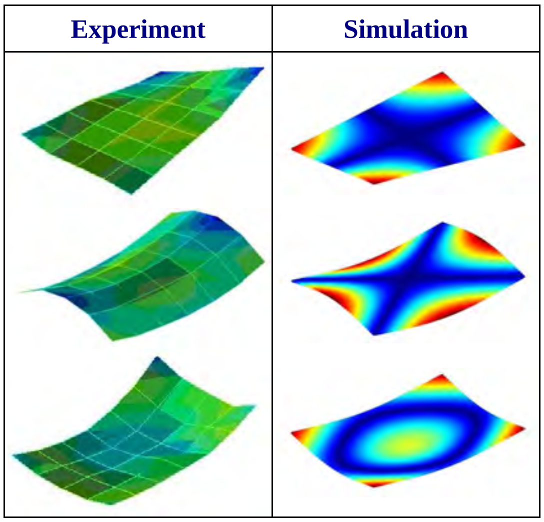

Figure 2. Dynamic modulus identification.

Like shown on Figure 2, the measured frequencies (and associated mode shapes) are compared with computed resonant frequencies of a numerical parameter model of the test plate. Starting from an initial guess, the parameters of the numerical model are tuned until the computed resonant frequencies match the measured ones. The obtained elastic properties are homogenised over the plate surface, and hence suitable as averaged input values in finite element models to perform stiffness and strength calculations for orthotropic materials like sandwich structures with steel skins.

2. Resonalyser Non Destructive Testing

In plane stress conditions, the constitutive law for the elastic behaviour of orthotropic materials reduces to

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{E_1} & -\dfrac{\nu_{12}}{E_2} & 0 \\ -\dfrac{\nu_{21}}{E_1} & \dfrac{1}{E_2} & 0 \\ 0 & 0 & \dfrac{1}{G_{12}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix} \qquad (01)$$

where the in-plane elastic properties $\{E_1, E_2, G_{12}, \nu_{12}\}$ can be determined by the Resonalyser method, comparing the numerical results from a modal analysis with measured resonant frequencies. This tuning technique is a Bayesian parameter estimation method, based on the sensitivities of the resonant frequencies for parameter changes.

The inverse procedure of the Resonalyser test can only yield good results if the numerical model is controllable, and if the elastic properties can be observed through the measured data [03]. This requires that in the selected series of frequencies, at least one of the frequencies varies significantly for variations of each of the elastic properties. It can be shown [04] that this requirement is fulfilled if the aspect ratio a/b (length versus width) of the test plate is close to

$$\frac{a}{b} = \sqrt[4]{\frac{E_1}{E_2}} \qquad (02)$$

A plate with an aspect ratio satisfying (02) is called a ‘Poisson’ test plate, and shows a predictable sequence of mode shapes, like shown on Figure 2. The first three resonance modes correspond to a torsional, an anticlastical (saddle) and a synclastical (breathing) mode.

The name ‘Poisson test plate’ has been chosen based on the observation that the resonance frequencies of the anticlastic and synclastic mode shapes are particularly sensitive for changes of the Poisson’s ratio of the material. A hypothetical material with $\nu_{12} = 0$ would make the frequencies of both modes coincide. A value $\nu_{12} \neq 0$ gives rise to an increasing eigenvalue

$$\lambda_b = \left(2\pi f_b\right)^2 \qquad (03)$$

for the breathing mode, and a decreasing eigenvalue λ_s for the saddle mode. Using an empirical formula [05], it is possible to relate the Poisson's ratio ν_{12} with the eigenvalues of the saddle and the breathing modes:

$$\nu_{12} \propto \frac{\lambda_b - \lambda_s}{\lambda_b + \lambda_s} \qquad (04)$$

The magnitude of the torsional eigenvalue λ_t is almost exclusively determined by the shear modulus G_{12}, and can be expressed as

$$\lambda_t \propto G_{12} \frac{t^3}{M\,a\,b} \qquad (05)$$

with {a, b, t} the plate dimensions, and M the mass of the test plate.

3. Vibro-acoustic measurements

A semi-anechoic test lab is installed at OCAS for dedicated vibro-acoustic measurements. The lab is built as a box within a box, and has a total volume of 80 m³. The test chamber is acoustically isolated, and insensitive to vibration thanks to its well engineered mounting system. The concrete walls (with a thickness of 380 mm) and the ceiling are lined with sound absorbing material, while the floor is cladded with reflective planes. This semi-anechoic test lab allows for a wide range of vibrational tests:

- Order analysis
- Modal analysis
- Dynamic analysis
- Oberst damping test
- Vibration measuremetns
- Grindosonic stiffness measurements

and special acoustic experiments to measure sound power, sound mapping and insulation.

The setup for the resonalyser experiments, shown in Figure 3, includes a suspension frame for the rectangular test plate, an acoustic excitation, an accelerometer, a signal conditioning unit, a data acquisition system and a personal computer. The test plate is suspended on the frame with elastic threads, simulating completely free boundary conditions.

Figure 3. Experimental test facility

The test plate is then excited by an impulse with a hammer. The vibration amplitude of the plate as a function of time is monitored by the accelerometer and captured by the data acquisition system.

The resonance frequencies of the plate in the band of interest are detected by taking the Fast Fourier Transform (FFT) of the signal. Then a sinusoidal signal with a frequency coinciding with each resonance frequency is sent to the loudspeaker, acoustically exciting the test plate. The decay of the signal allows extracting the modal damping ratios and the mode shapes associated wit the resonance frequencies.

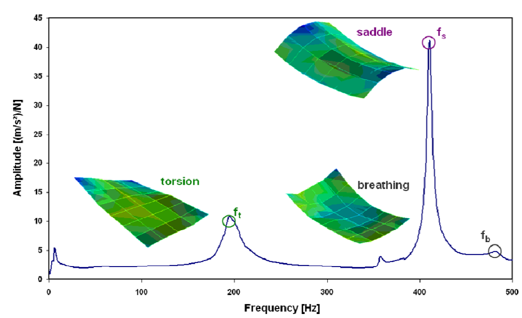

Figure 4. Frequency response function + mode shapes

Figure 4 shows a typical Frequency Response Function (FRF). Such a signal is a frequency domain function expressing the ratio between the response signal (output) and the reference signal (input). The peaks in the FRF indicate that low input levels generate high response levels, which reveals resonance frequencies. The corresponding mode shapes are shown as well.

For a composite plate with length a = 200 mm, width b = 143 mm, thickness t = 3.78 mm and total mass M = 0.191 kg, the measured eigenfrequencies and corresponding damping coefficients are listed in Table 1:

Table 1. Resonalyser plate test results

Mode shape	Frequency [Hz]	Damping [-]
Torsion	218.84	0.588
Saddle	438.43	0.617
Breathing	496.79	0.280

4. Numerical implementation

The Structural Mechanics Module of COMSOL Multiphysics is used to calculate the mode shapes for the test plate of Table 1 with 'dummy' properties. For these material properties, an 'educated' initial guess is used, e.g. based on micromechanical modelling [06]. The measured frequencies of the test plate are compared with the computed frequencies to identify the in-plane orthotropic properties of the material. A detailed flowchart of the Resonalyser procedure is given in Figure 5, showing that COMSOL Multiphysics and MatLab are closely connected in solving this reverse engineering problem.

4.1 Finite element model in COMSOL

The Structural Mechanics Module of COMSOL Multiphysics is used to calculate the mode shapes for a test plate with fixed length a, width b, thickness t and mass M. The dynamic equilibrium equations of a freely vibrating (Love-Kirchoff) plate can be written as an algebraic eigenvalue problem:

$$\left(K_{ij} - \lambda M_{ij}\right)\{\Phi\} = \{0\} \qquad (06)$$

with mass matrix

$$M_{ij} = \rho t \frac{a b}{4} A_{ij} \qquad (07)$$

and stiffness matrix

$$\begin{aligned} K_{ij} = {} & \frac{4b}{a^3} D_{11} B_{ij} + \frac{4a}{b^3} D_{22} C_{ij} \\ & + \frac{16}{a b} D_{66} E_{ij} + \frac{4}{a b} D_{12} F_{ij} \\ & + \frac{8}{a^2} D_{16} G_{ij} + \frac{8}{b^2} D_{26} H_{ij} \end{aligned} \qquad (08)$$

where A_{ij}, B_{ij}, C_{ij}, E_{ij}, F_{ij}, G_{ij} and H_{ij} are constant matrices, containing partial derivatives of the shape functions used for the Rayleigh-Ritz approximation [07] of the vibrational behaviour of the plate. The plate rigidities D_{ij} can be expressed in terms of the orthotropic engineering constants:

$$D_{11} = \frac{E_1 t^3}{12\left(1-\nu_{12}\nu_{21}\right)} ; D_{12} = \nu_{12} D_{22} \qquad (09)$$

$$D_{22} = \frac{E_2 t^3}{12\left(1-\nu_{12}\nu_{21}\right)} ; D_{66} = \frac{G_{12} t^3}{12} \qquad (10)$$

The solution of (06) yields the eigenvalues λ_i and the corresponding modeshapes $\{\Phi\}^{(i)}$. The eigenfrequencies are calculated from and stored as f_{fem} = `eigfreq_smsld(i)`. In Figure 2, the modeshapes calculated by COMSOL are compared with the measurements from the modal analysis.

4.2 Optimisation kernel in MatLab

The in-plane elastic constants are stored in a parameter column

$$\{p\} = \begin{Bmatrix} E_1 \\ E_2 \\ \nu_{12} \\ G_{12} \end{Bmatrix} \qquad (11)$$

For each iteration, the difference

$$\Delta f(p) = \left\{ f_{\exp} - f_{fem} \right\} \qquad (12)$$

between the measured and the predicted eigenfrequencies is calculated.

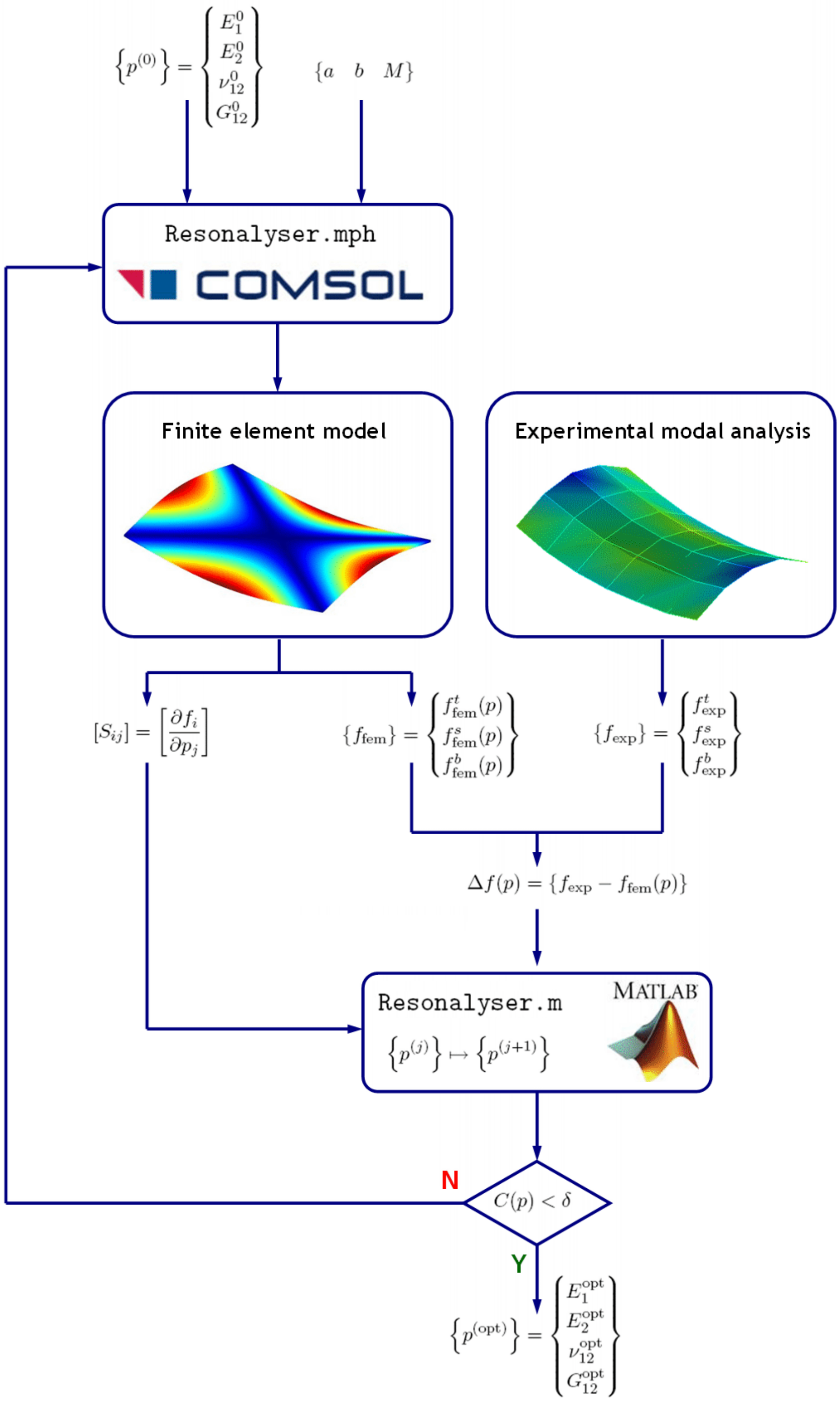

Figure 5. Flowchart of the Resonalyser method when implemented in COMSOL and MatLab

Updating the vector (11) with the elastic constants is performed in the MatLab `Resonalyser.m` script. The optimisation algorithm is based on minimizing the cost function [08]

$$C(p)=\{\Delta f(p)\}^T\left[W^{(f)}\right]\{\Delta f(p)\} + \left\{p^{(0)}-p\right\}^T\left[W^{(p)}\right]\left\{p^{(0)}-p\right\} \tag{13}$$

in which C(p) is a $\mathbb{R}^{NP} \mapsto \mathbb{R}$ cost function yielding a scalar value, NP = 4 is the number of material parameters in (11), and $\{p^{(0)}\}$ is an (NPx1) vector that contains the initial guesses. The measured frequencies $\{f_{exp}\}$ and the computed frequencies $\{f_{fem}\}$ both have dimensions NFx1, with NF = 3 the number of frequencies studied. The choice and role of the weighting matrices $[W^{(f)}]$ and $[W^{(p)}]$ are discussed in [09].

The cost function C(p) has a minimum value for the optimum parameter set $\{p^{(opt)}\}$. Updating for the {p} vector in iteration step (j+1) can be expressed by a recurrence formula

$$\left\{p^{(j+1)}\right\}=\left\{p^{(j)}\right\} + \left[W^{(p)}+\left(S^{(j)}\right)^T W^{(f)} S^{(j)}\right]^{-1} \times S^{(j)} W^{(f)}\left\{f_{\exp}-f_{fem}\left(p^{(j)}\right)\right\} \tag{14}$$

where *S* is the sensitivity matrix containing the partial derivatives of the numerical frequencies with respect to the elements of the parameter column. This iteration procedure ends when convergence of {p} is reached, i.e. when

$$C(p)<\delta \tag{15}$$

with δ a predescribed tolerance, or when the maximum number of iterations j = N is reached. The values $\{p^{(opt)}\}$ of the engineering constants in the last iteration cycle provide a good approximation of the averaged mechanical properties of the full test plate.

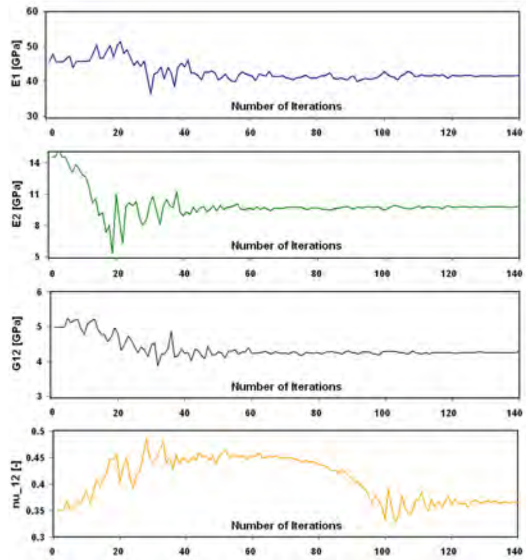

Figure 6. Iteration history for in-plane properties

The iteration history for the in-plane properties of the test plate under study is shown on Fig. 6. For E_2 and G_{12}, the optimum solution is found after 60 evaluations of the cost function C(p). After 140 iterations, the convergence criterion (15) is met for all elastic properties. These final results are listed in Table 2:

Table 2. Resonalyser plate simulation results

		Initial	Optimum
E_1	[MPa]	45 800	**41 958**
E_2	[MPa]	15 500	**9 810**
ν_{12}	[-]	0.35	**0.365**
G_{12}	[MPa]	5 000	**4 287**

This section clearly shows that the smooth interface between COMSOL and MatLab allows identifying the material properties of complex composites by means of reverse engineering schemes. COMSOL Multiphysics provides both the capability to calculate the modeshapes of orthotropic plates, and to interact with the MatLab optimisation kernel.

The screenshot on Figure 7 proves the close interaction between these software packages, bringing out the best of both worlds.

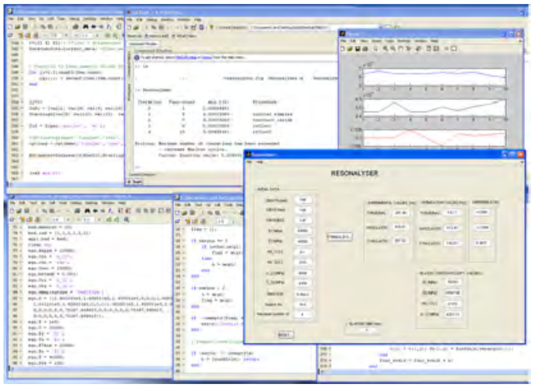

Figure 7. Close interaction COMSOL - MatLab

5. Experimental validation

The Resonalyser method yields averaged properties that describe the mechanical behaviour of composite structures. In order to validate the results listed in Table 2, traditional mechanical tests were performed. Quasi-static tensile tests were conducted to determine the Young's moduli E_1 and E_2 and the corresponding strength values X_T and Y_T. Using two strain gauges, the in-plane contraction coefficient ν_{12} can be derived from these experiments as well. The shear modulus G_{12} and strength *S* were derived from rail shear tests. The results of the mechanical characterization are summarized in Table 3 and compared with the Resonalyser (non destructive) test results:

Table 3. Experimental validation

		Mechanical	**Resonalyser**
E_1	[MPa]	36 400	41 958
E_2	[MPa]	8 500	9 810
ν_{12}	[-]	0.32	0.365
G_{12}	[MPa]	3 190	4 287
X_T	[MPa]	700	
Y_T	[MPa]	7.2	
S	[MPa[	28.2	

Table 3 indicates that the Resonalyser test method tends to overestimate the values measured in the experiments.

Indeed, the Resonalyser method will typically give higher stiffness values because

- It is a **non destructive** (NDT) method. When performing traditional mechanical testing, sample preparation can already induce microscopic damage, lading to a lower stiffness and strength. Moreover, the effects of slip are avoided when performing NDT, again resulting in higher (and more accurate) estimates for the elastic constants.
- It measures the **global** (homogenized) properties of a composite structure, rather than the local stiffness of a small specimen, which can exhibit quite some scatter.

To prove this point, the Resonalyser procedure was performed with the experimental results as initial values. Table 4 indeed reveals that the Resonalyser procedure will provide higher values for the elastic constants:

Table 4. Resonalyser with experiment as initial value

		Experiment	**Optimum**
E_1	[MPa]	36 400	**39 254**
E_2	[MPa]	8 500	**10 541**
ν_{12}	[-]	0.31	**0.332**
G_{12}	[MPa]	3 335	**4 297**

The COMSOL implementation of the Resonalyser procedure ("OCAS") was compared with published experimental and numerical results ("SOL", cfr. [10]). The results of this benchmark, shown on Figure 8, again confirm that the Resonalyser method will overestimate the experimental values. In addition, the OCAS procedure is seen to perform more accurately than the algorithm proposed in [10].

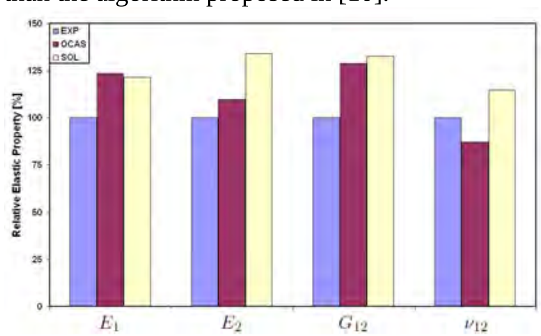

Figure 8. Benchmarking the Resonalyser procedure

6. Conclusions

In this paper, the Resonalyser method was introduced as a material identification technique following a reverse engineering scheme. It is a non destructive test methodology to measure the average elastic properties of orthotropic materials. The in-plane elastic properties $\{E_1, E_2, \nu_{12}, G_{12}\}$ were determined by a dynamic modulus identification using resonant frequencies. Both the determination of the real part (elastic constants) and the imaginary part (damping behaviour) are based on the measurement of the vibrational response of a rectangular test plate, submitted to a controlled excitation. COMSOL Multiphysics is instrumental in the proposed reverse engineering scheme, as it provides both the calculation of mode shapes and the interfacing with the optimisation kernel of MatLab.

7. References

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