Finite Element Modeling of Coupled Heat and Mass Transfer of a Single Maize Kernel Based on Water Potential Using COMSOL Multiphysics Simulation
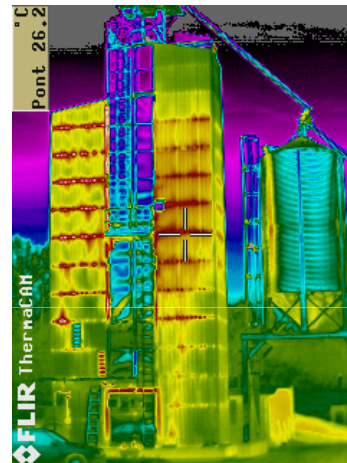
Authors:

Attila J. Kovács, E. Lakatos, G. Milics and M. Neményi

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Main research area of our Institute is grain drying since the 1980's.

Testing of grain dryers

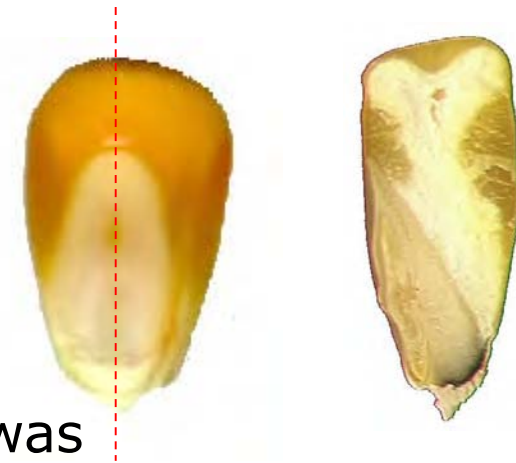Thermographic image of the dryer

Experimental convective drying tunnel

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Besides experimental research drying modeling of a single kernel was initiated.

For example, the **mass diffusion** relationship was determined for different maize hybrids:

$$D = e^{a+bX+cT^{-1}}$$

where: D   =   moisture diffusion coefficient;

X   =   kernel average moisture content (dry basis);

T   =   average temperature of kernels;

a, b, c   =   constants.

(These constants for the Pioneer 3780 hybrid were found: $a = -3.1411$; $b = 3.2159$; $c = -6696.6$)

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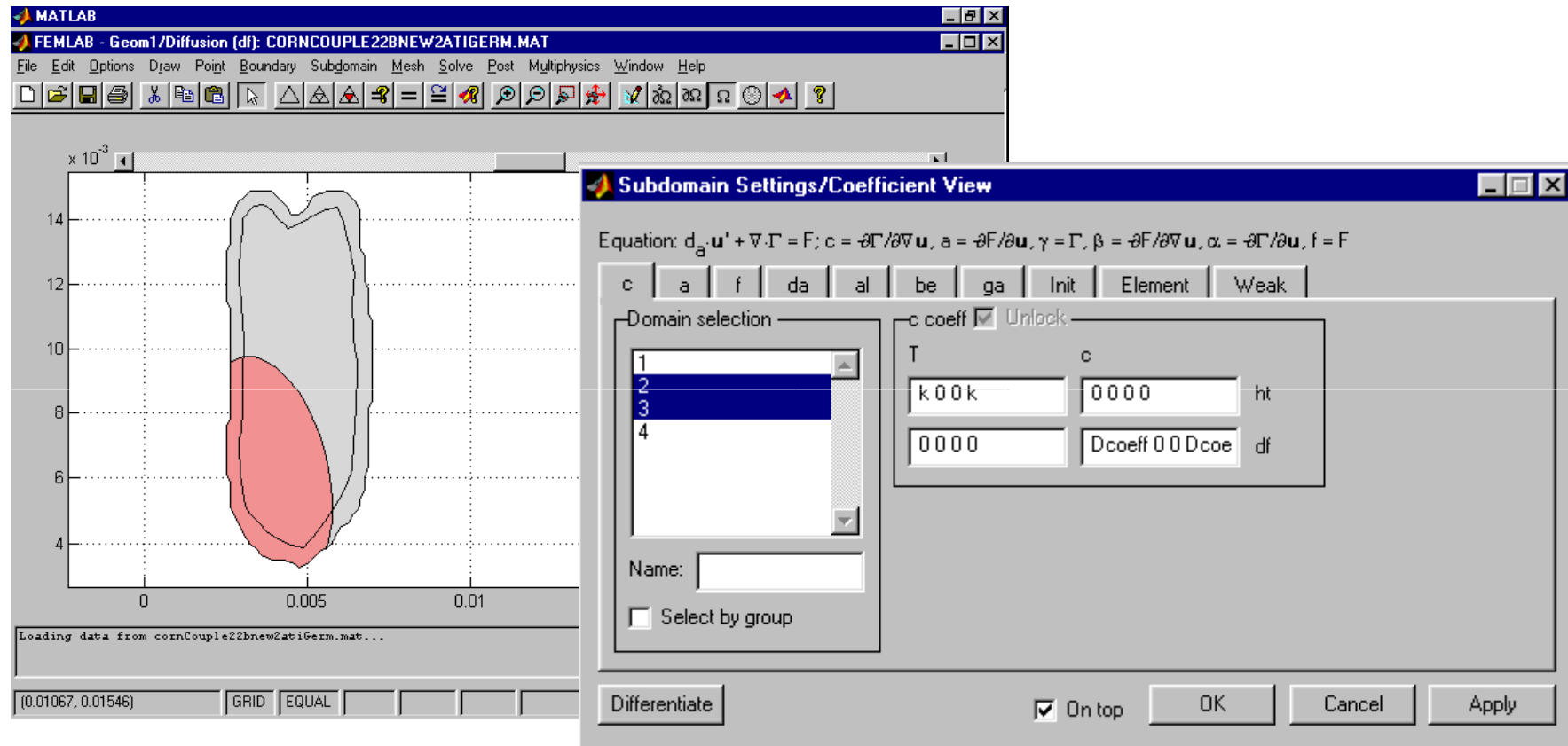Tel.: +36 96 566 635 Fax: +36 96 566 641 E-mail: kovacsaj@mtk.nyme.hu

Based on the measurements mathematical modeling studies of drying were carried out using Finite Element method:

$$\rho \cdot c \frac{\partial T}{\partial t} = k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + L \cdot \rho \frac{\partial X}{\partial t}$$

$$\frac{\partial X}{\partial t} = D(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 X}{\partial y^2})$$

for heat transfer

for mass transfer

where: X is moisture content d.b. [kg/kg];
    ρ is density [kg/m^3];
    T is temperature [K];
    K is thermal conductivity [W/mK];
    L is latent heat of vaporisation
      of water [J/kg].

D is diffusion coefficient [m^2/s];
c is specific heat [J/kgK];
t is time [s];
x,y are directions;

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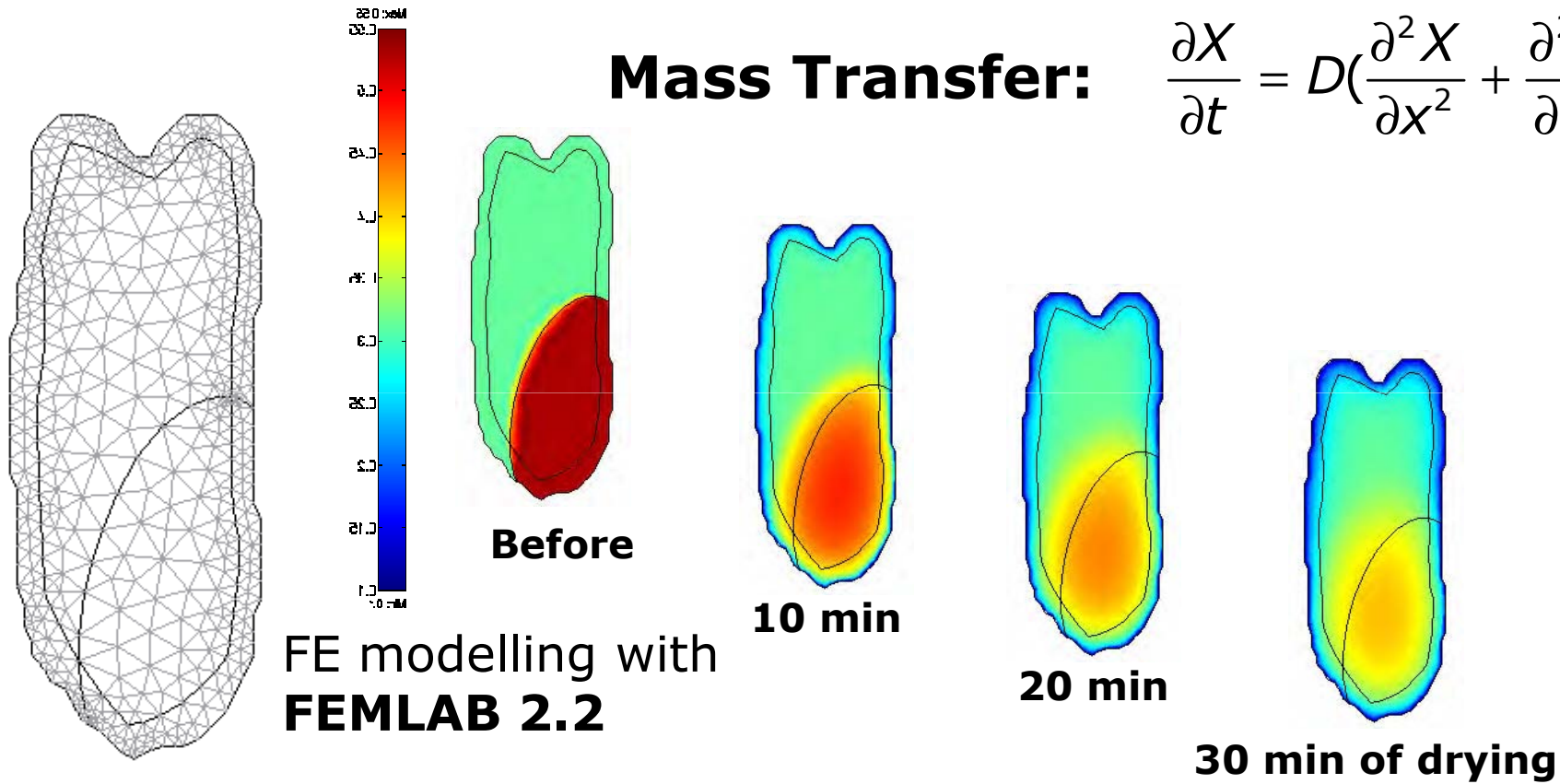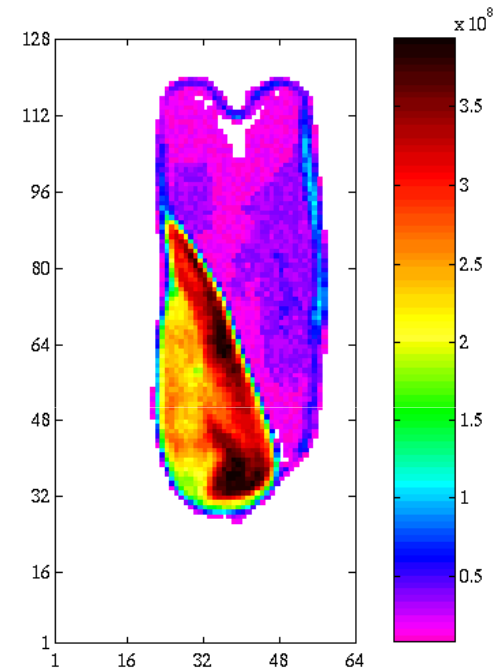Tel.: +36 96 566 635 Fax: +36 96 566 641 E-mail: kovacsaj@mtk.nyme.hu

FE modelling with **COSMOS 1.71**
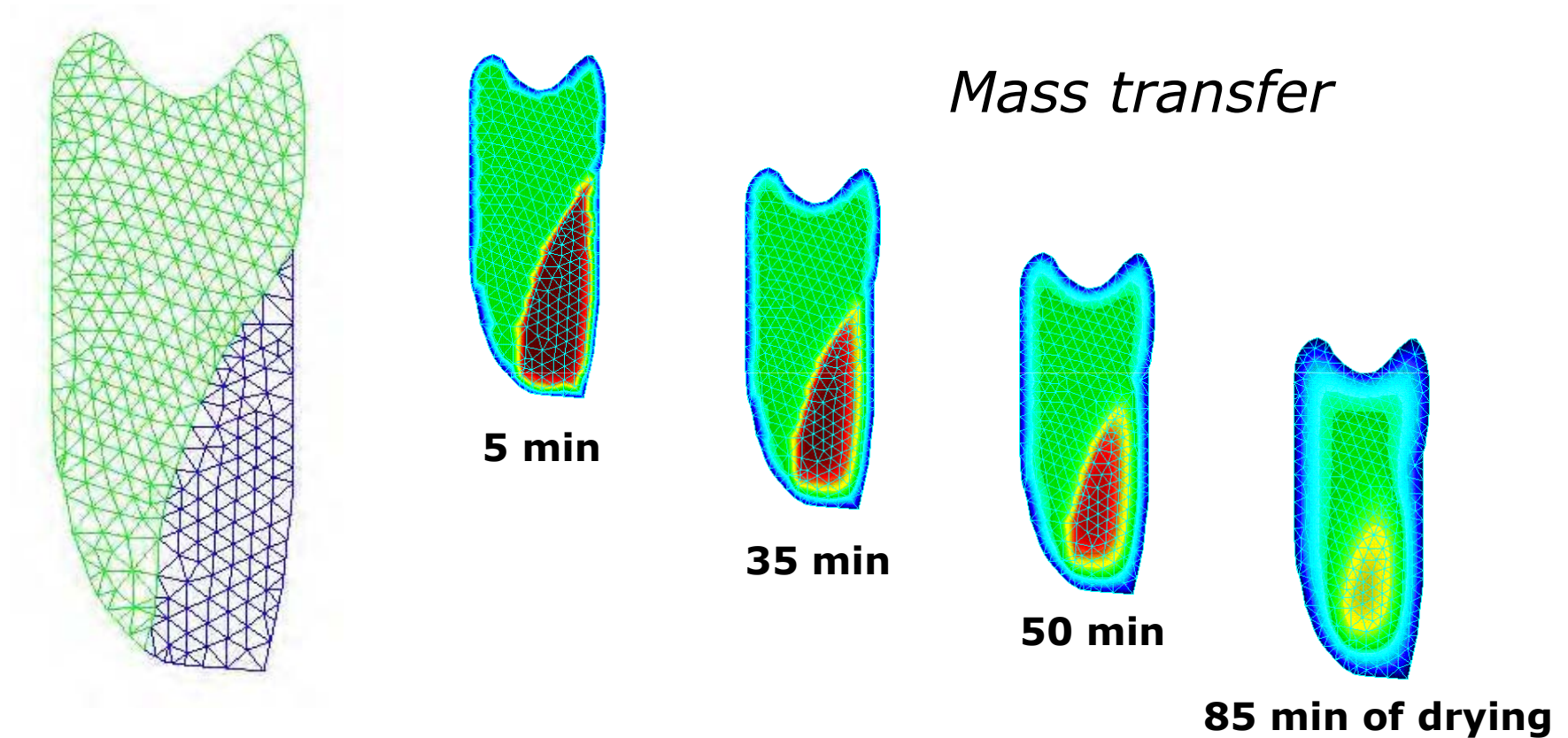
Mass transfer

5 min

35 min

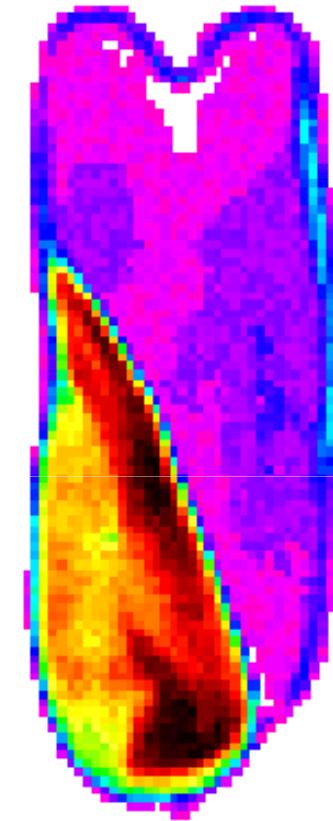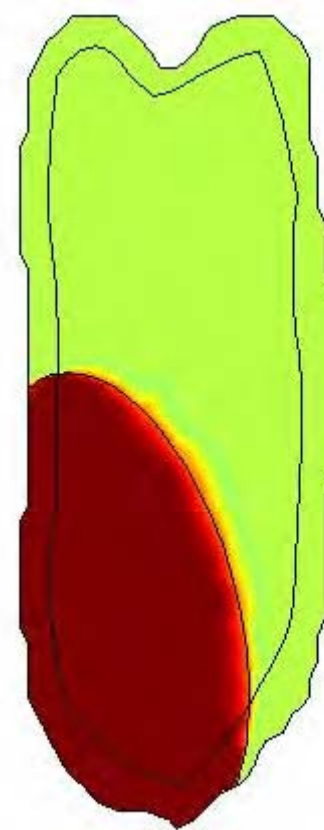**50 min**

85 min of drying

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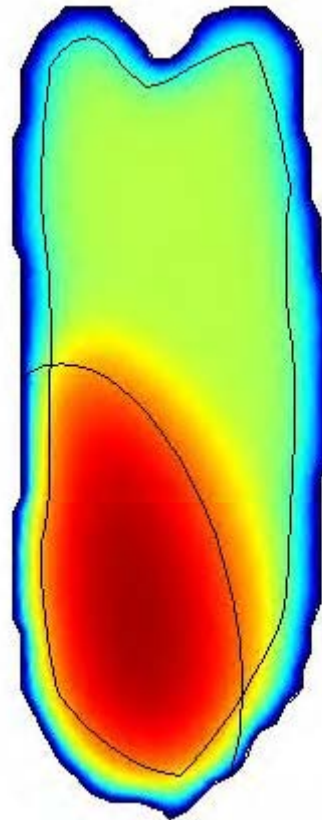# COMSOL MULTIPHYSICS simulation (FEMLAB 2.2 package)

University of West Hungary
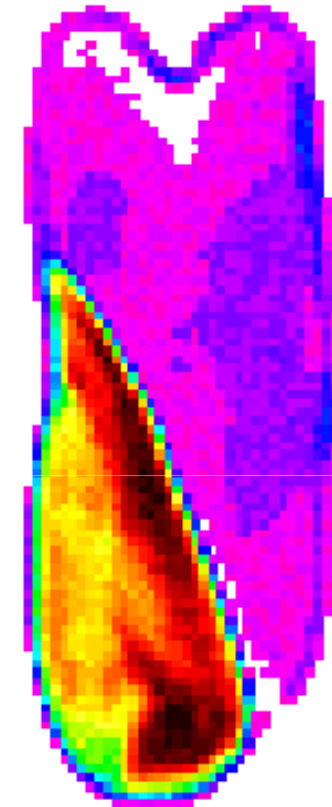**Faculty of Agricultural, Food and Environmental Sciences**
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Heat Transfer: $\rho \cdot c \dfrac{\partial T}{\partial t} = k(\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2}) + L \cdot \rho \dfrac{\partial X}{\partial t}$

Mass Transfer: $\dfrac{\partial X}{\partial t} = D(\dfrac{\partial^2 X}{\partial x^2} + \dfrac{\partial^2 X}{\partial y^2})$

Before

10 min

20 min

30 min of drying

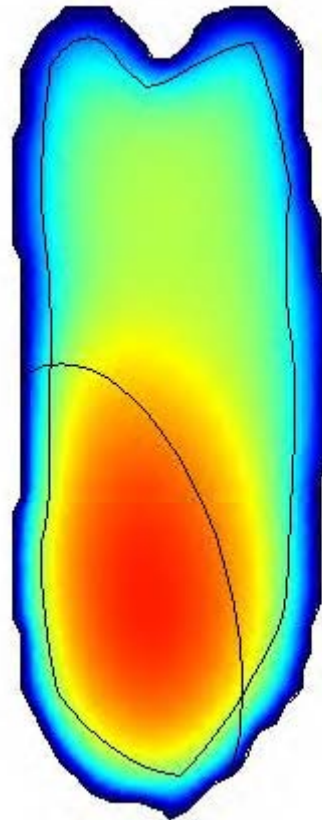FE modelling with
FEMLAB 2.2

University of West Hungary
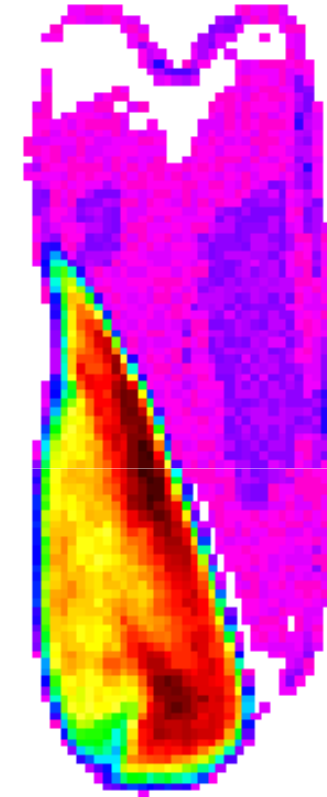**Faculty of Agricultural, Food and Environmental Sciences**
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Magnetic resonance imaging (MRI) studies were carried out to evaluate the accuracy of the FE models.

NMR magnet at BRUKER GMBH

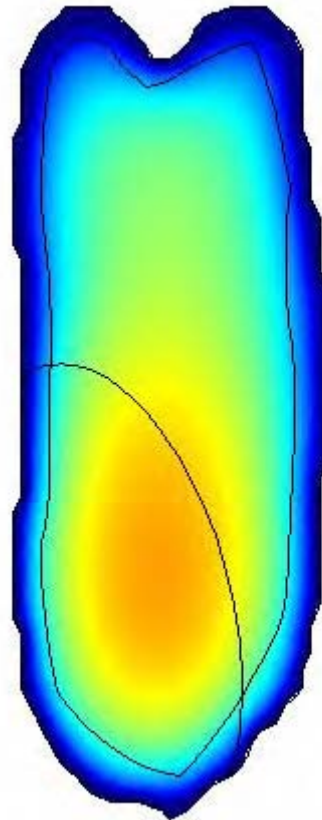**MRI image of a corn kernel before drying**

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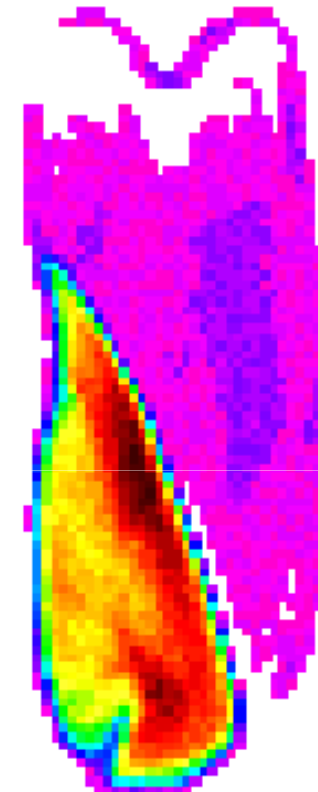Address: 2 Vár, Mosonmagyaróvár, H-9200 Hungary
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**MRI
measurement:**

FEMLAB model: the moisture content changes inside a single kernel during drying.

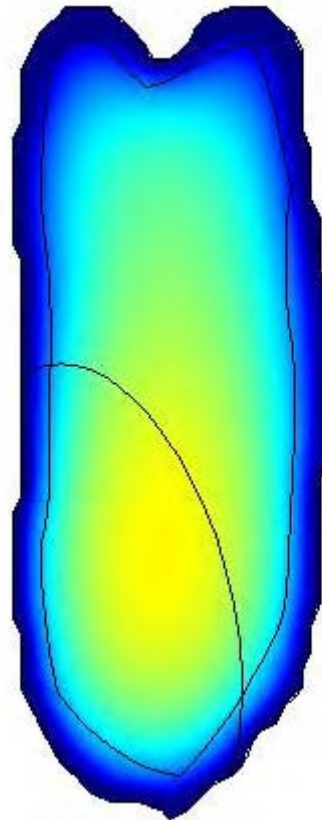- the model is based on the **m.c. gradients**

MRI measurement:

FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**

MRI measurement:

FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**

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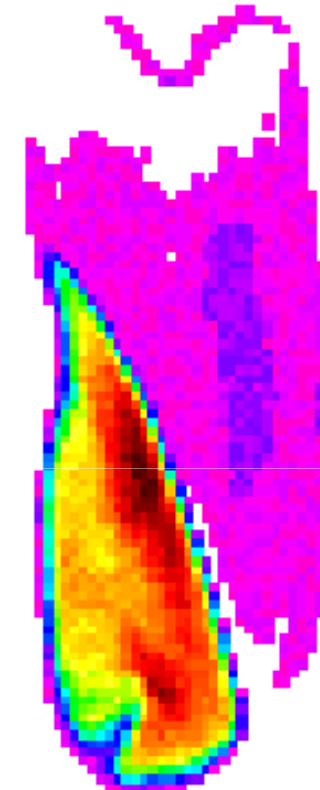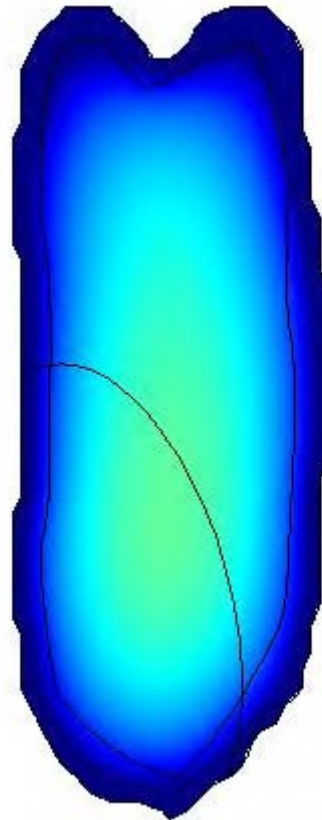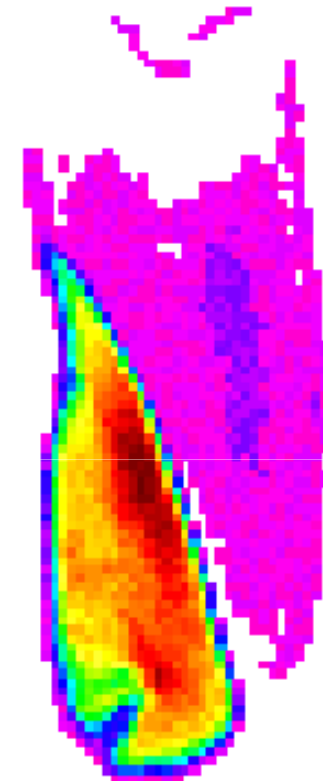**Institute of Biosystems Engineering**
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FEMLAB model: the moisture content changes inside a single kernel during drying.
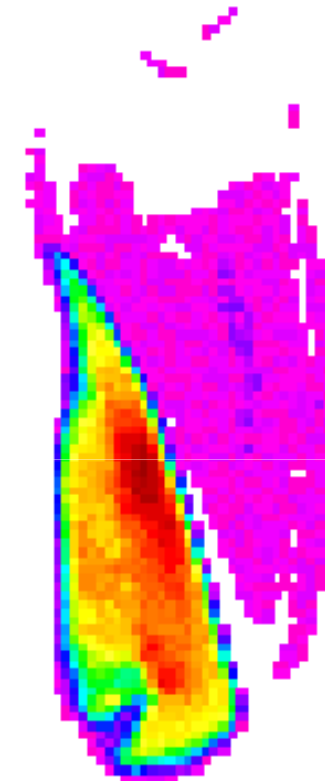
- the model is based on the **m.c. gradients**

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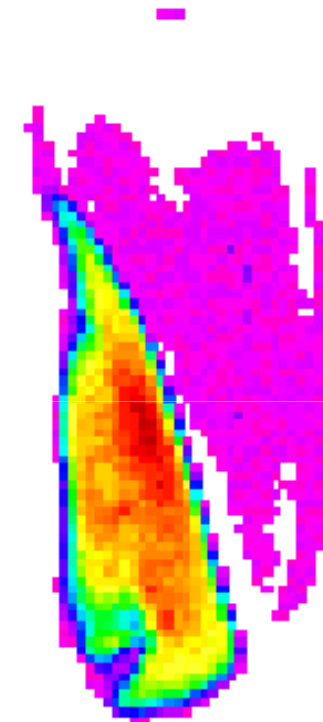**Institute of Biosystems Engineering**
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MRI measurement:

FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**

MRI measurement:

FEMLAB model: the moisture content changes inside a single kernel during drying.

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MRI measurement:
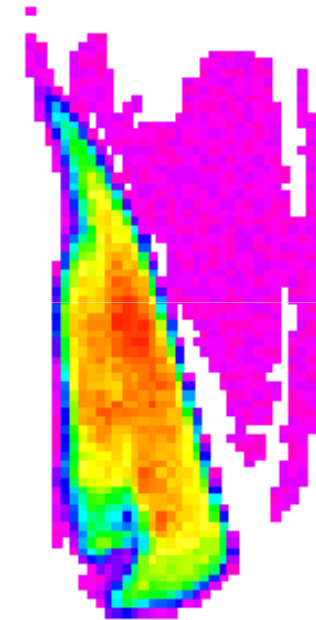
FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**

MRI measurement:

FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**

FEMLAB model: the moisture content changes inside a single kernel during drying.

- the model is based on the **m.c. gradients**

Moisture content of the particles vs. whole maize kernels (Florencia)
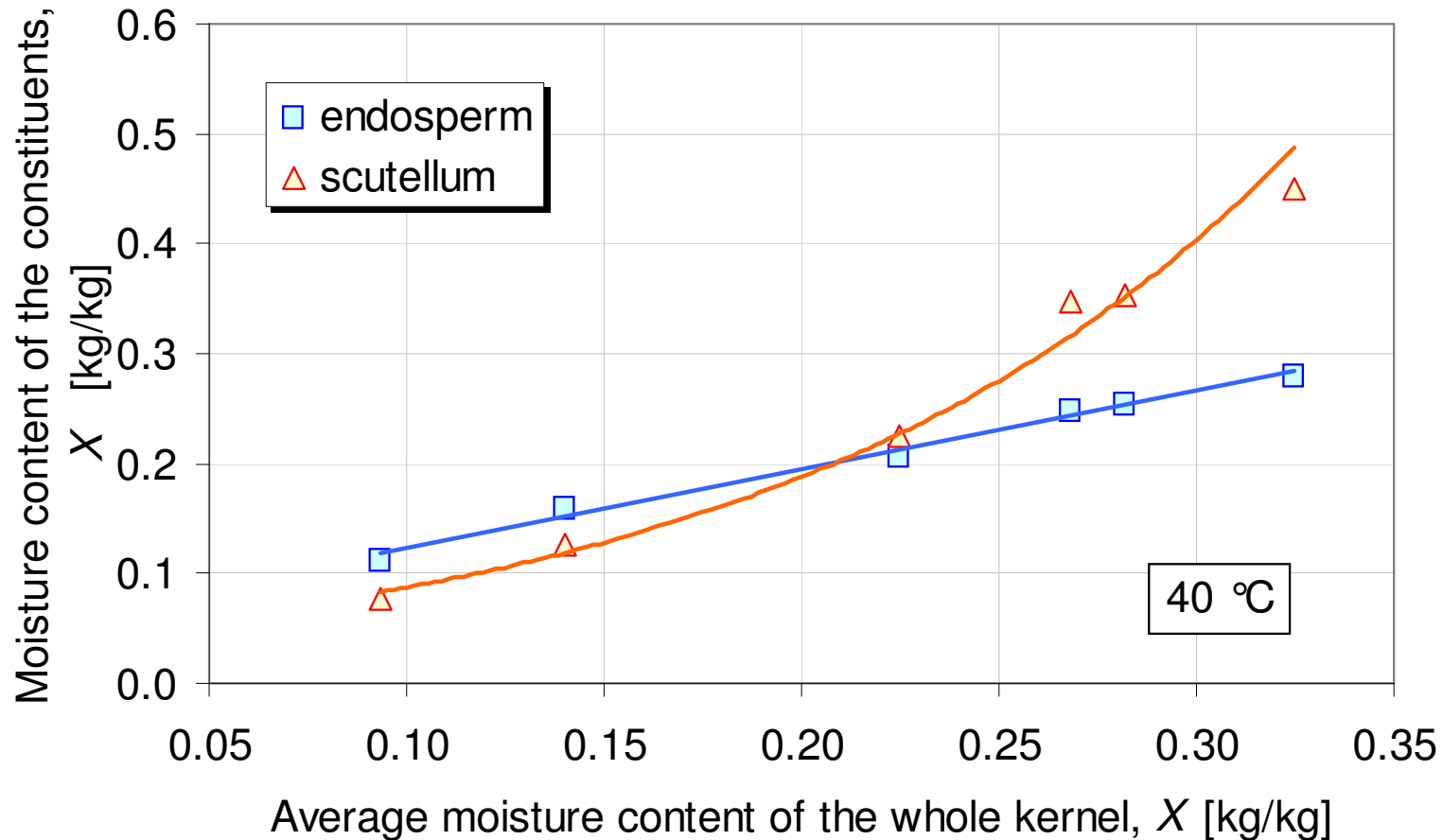
Considering capillary (water) potential

The modified **Luikov's model** of **mass transport**,

$$\frac{\partial X}{\partial \tau} = \mathrm{div}\, D \cdot \mathrm{grad}\, X$$

where: X is moisture content;
τ is time;
D is mass diffusion coefficient.

can be **rewritten**:

$$\frac{\partial X}{\partial \Psi}\frac{\partial \Psi}{\partial \tau} = \frac{\partial}{\partial x}\left[D\frac{\partial X}{\partial \Psi}\cdot\frac{\partial \Psi}{\partial x}\right] + \frac{\partial}{\partial y}\left[D\frac{\partial X}{\partial \Psi}\cdot\frac{\partial \Psi}{\partial y}\right] + \frac{\partial}{\partial z}\left[D\frac{\partial X}{\partial \Psi}\cdot\frac{\partial \Psi}{\partial z}\right]$$

and thus the equation **gives**:

$$\frac{\partial X}{\partial \Psi}\frac{\partial \Psi}{\partial \tau} = \mathrm{div}\, K \cdot \mathrm{grad}\,\psi$$

where: Ψ is capillary (water) potential;
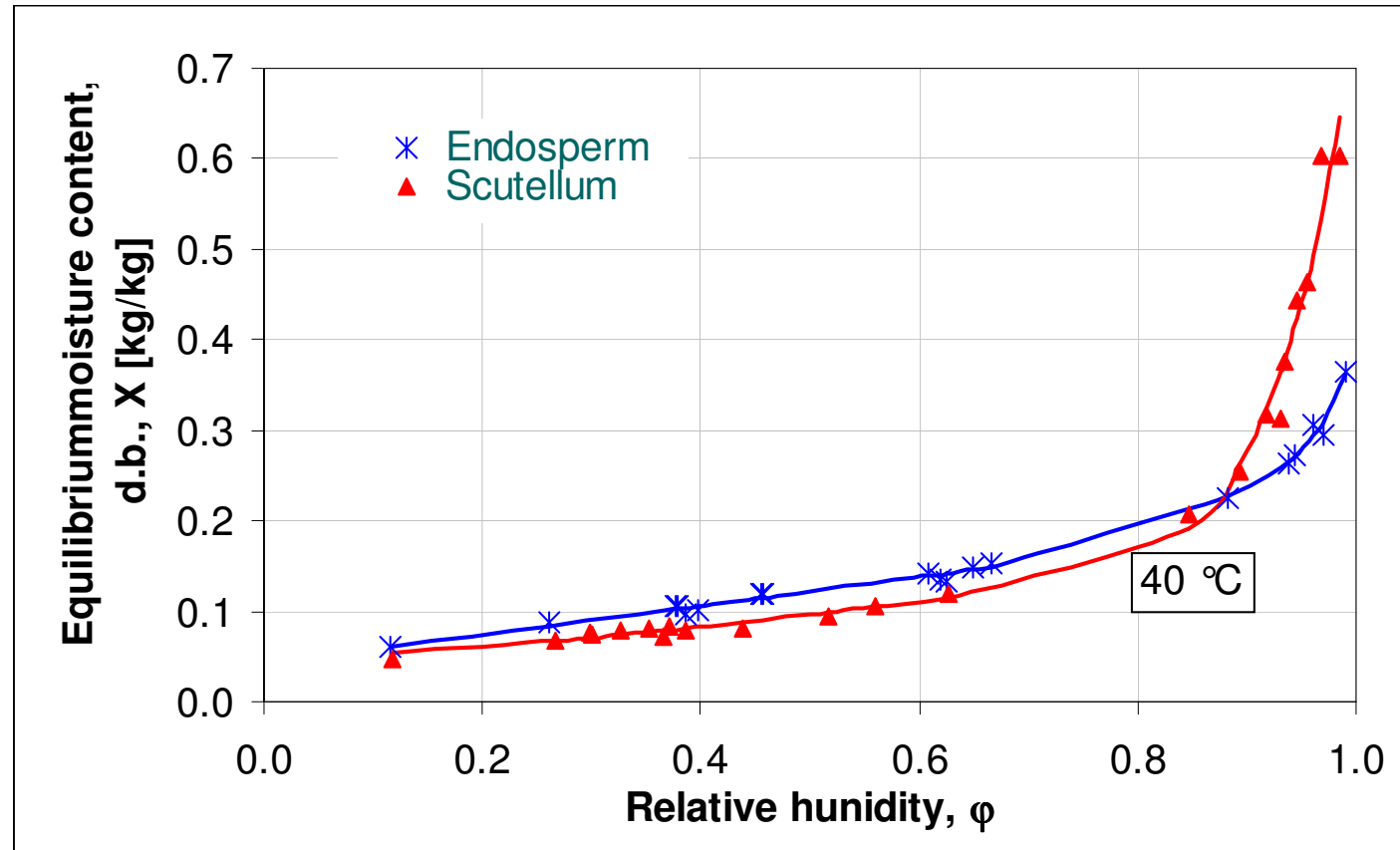K is hydraulic conductivity ($K = D\cdot\partial X/\partial Y$).

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Desorption curves of maize kernel's particles

ROTRONIC water activity meter

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Water potential curves of maize particles as a function of moisture content
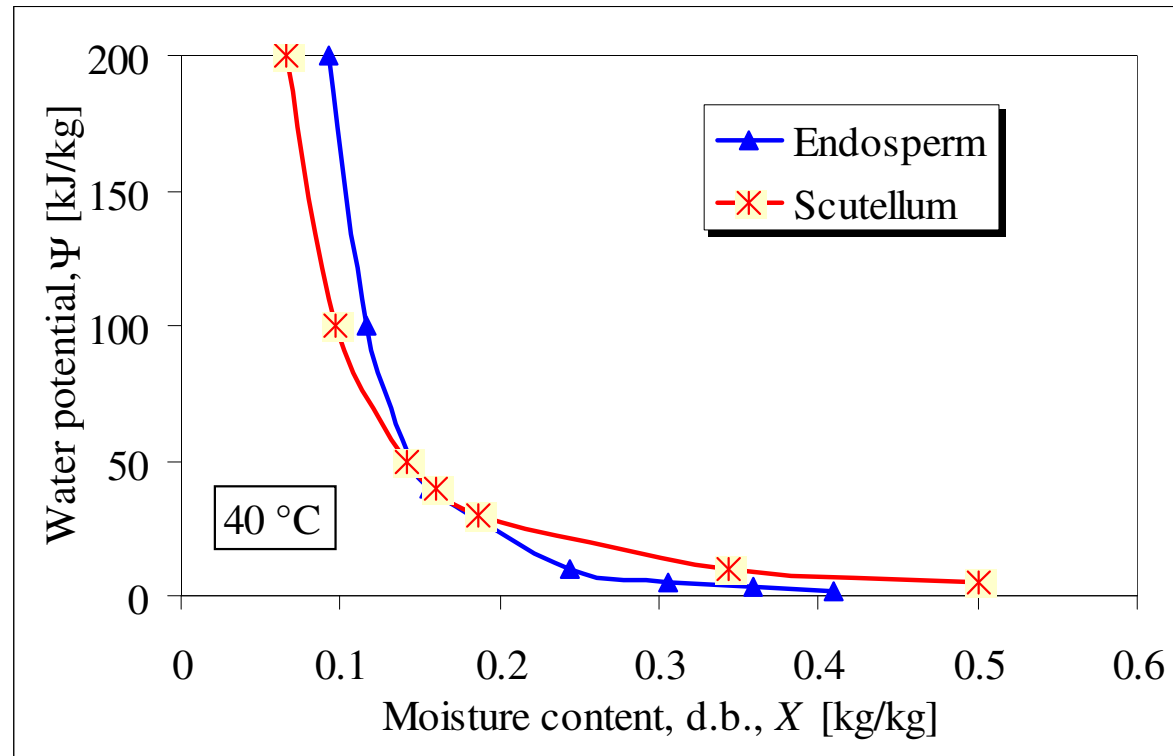
$$[\Psi]_X = \frac{R_U \cdot T}{m_V} \ln \varphi$$

where:

φ is the relative humidity of the air (decimal);

R_U is the universal gas constant;

m_v is the molecular weight of vapour;

T is the absolute temperature;

X is moisture content, d.b.

Boundary conditions

Neumann type:

$$\underline{n} \cdot (c\nabla u) + qu = qu_\infty$$

where:

u is stands for the water potential (Ψ);

$\underline{n}$ is the normal vector.

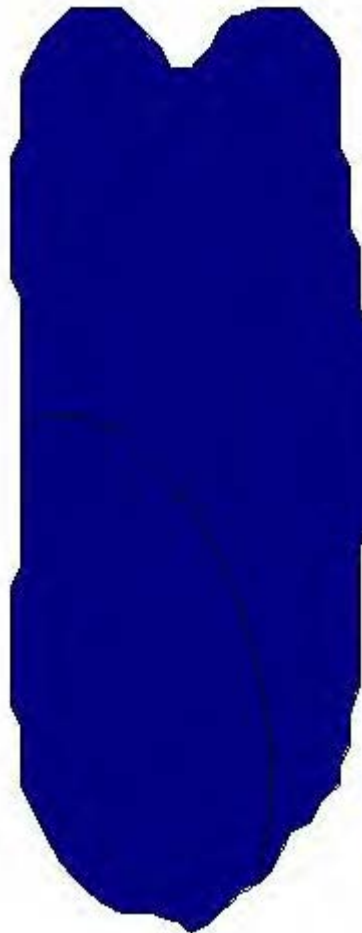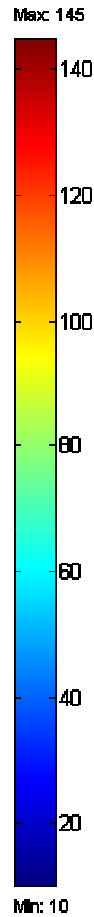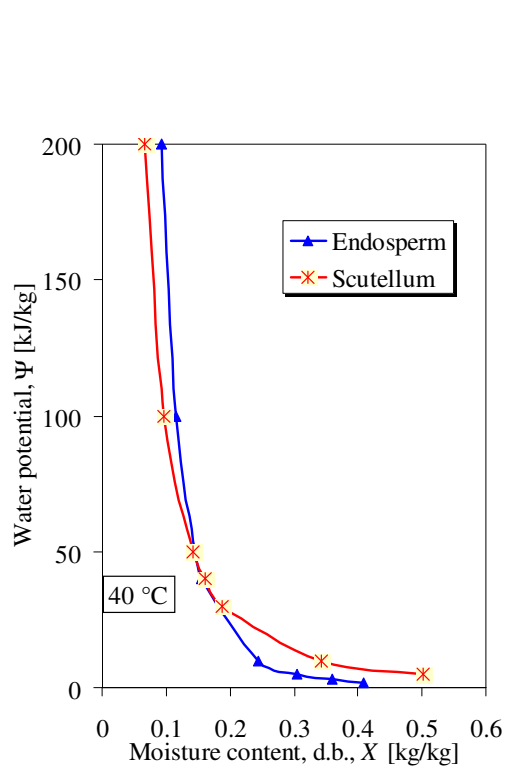$$d_a = \frac{\partial X}{\partial \psi}$$

Modelled Ψ changes

X changes based on grad Ψ

Water potential, [kJ/kg]

Moisture cotent, [kg/kg]

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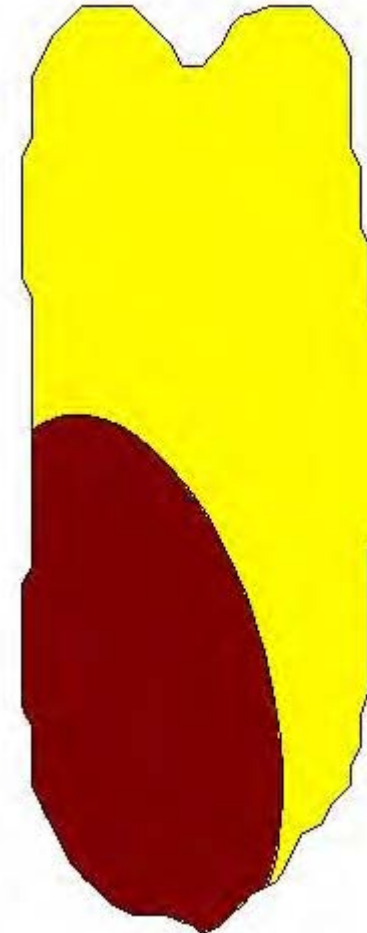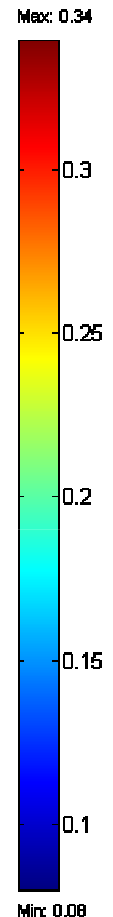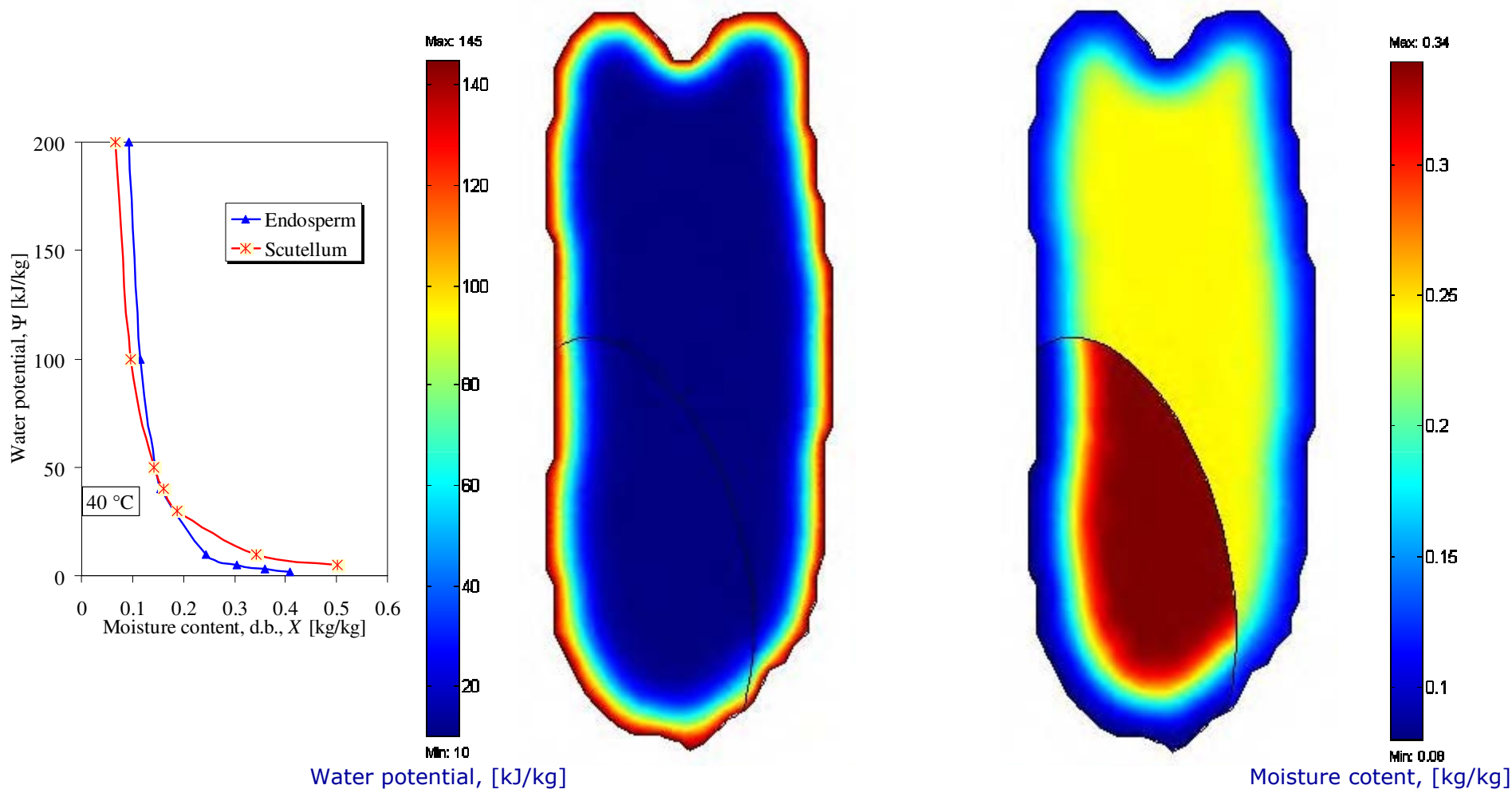**Institute of Biosystems Engineering**
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Modelled Ψ changes

X changes based on grad Ψ

Max 145

140

120

100

80

60

40

20

Min: 10

Water potential, [kJ/kg]

Max: 0.34

0.3

0.25

0.2

0.15

0.1

Min: 0.08

Moisture cotent, [kg/kg]

Water potential, Ψ [kJ/kg]

- Endosperm
- Scutellum

40 °C

Moisture content, d.b., X [kg/kg]

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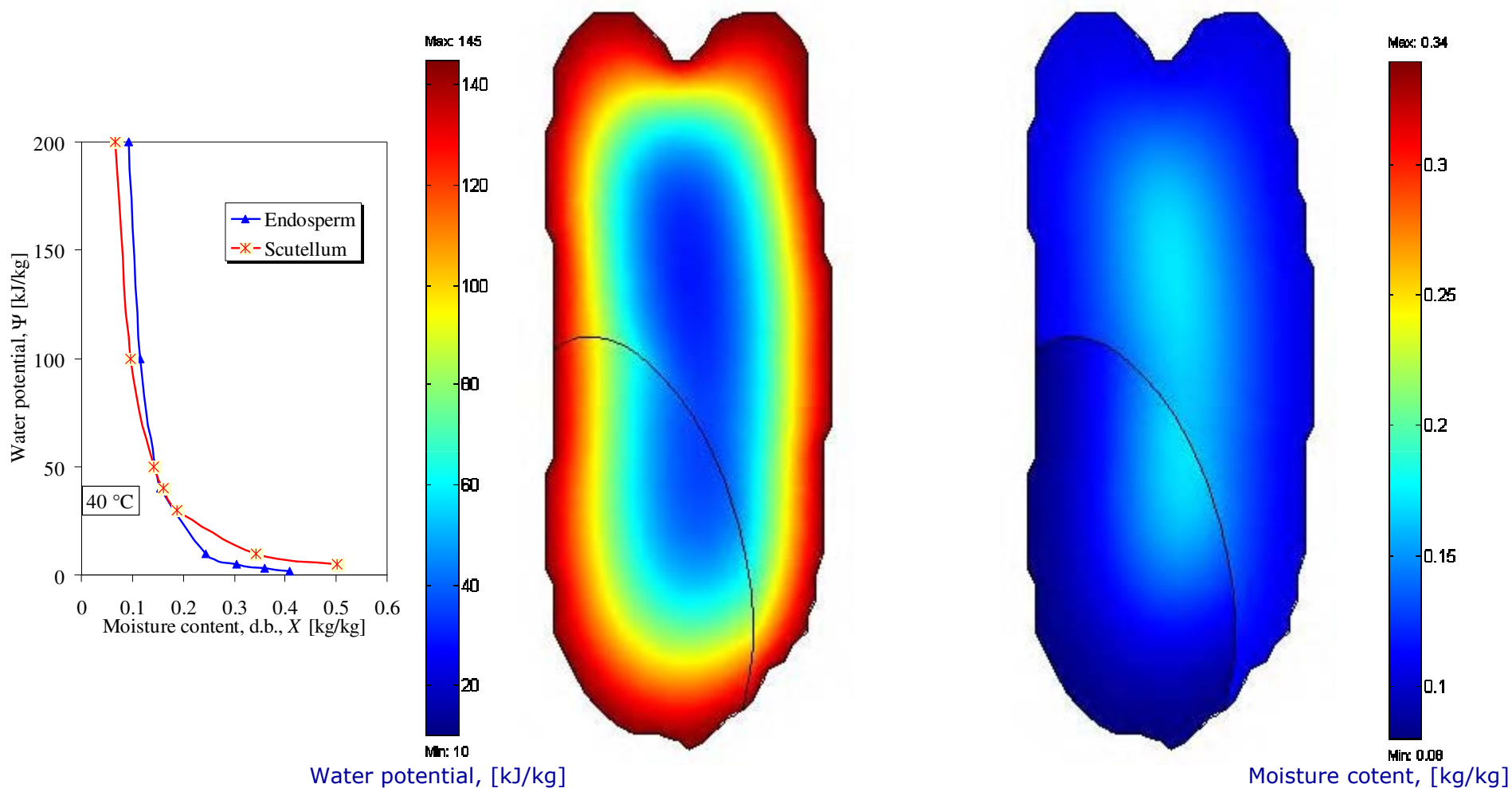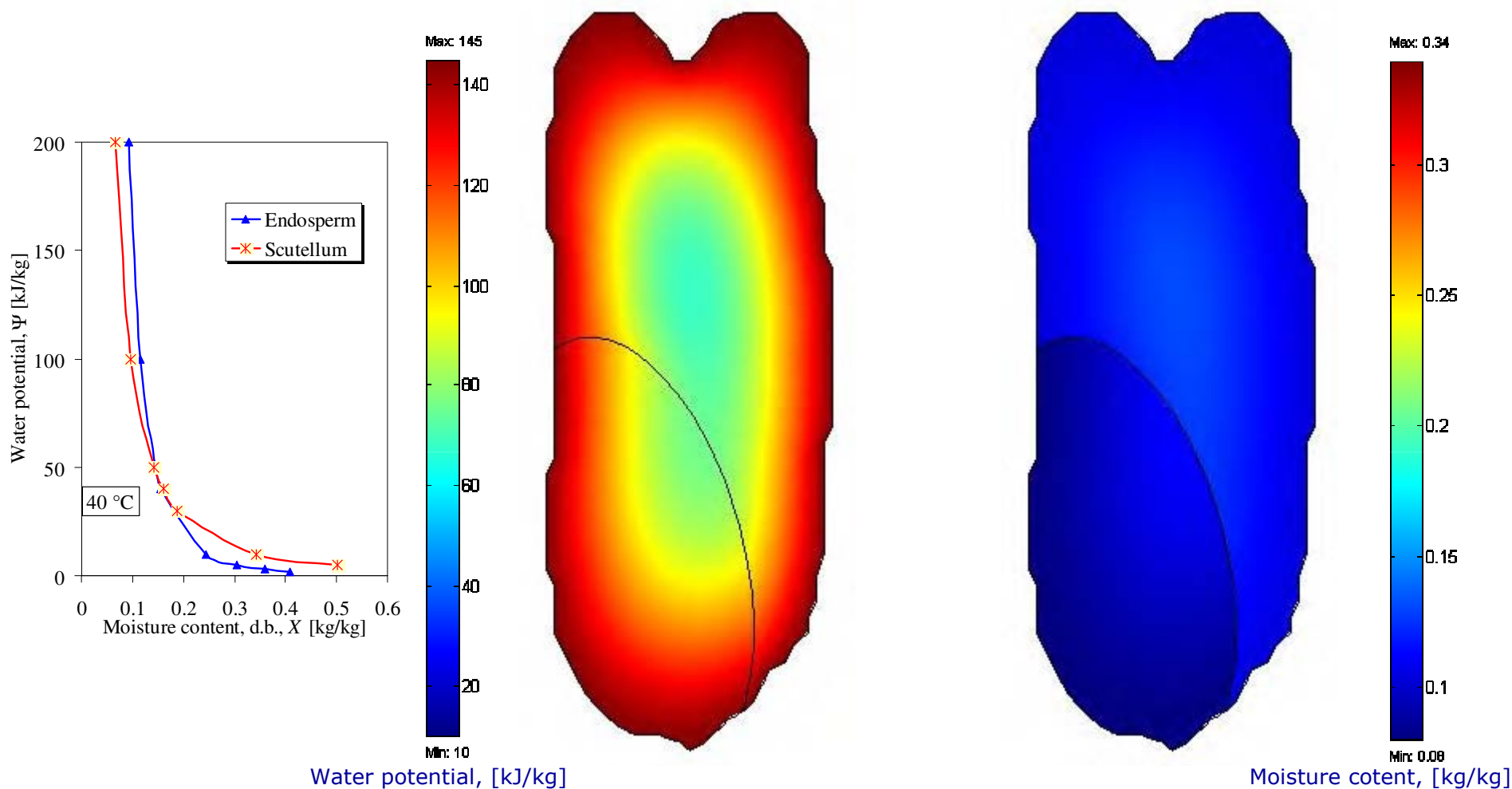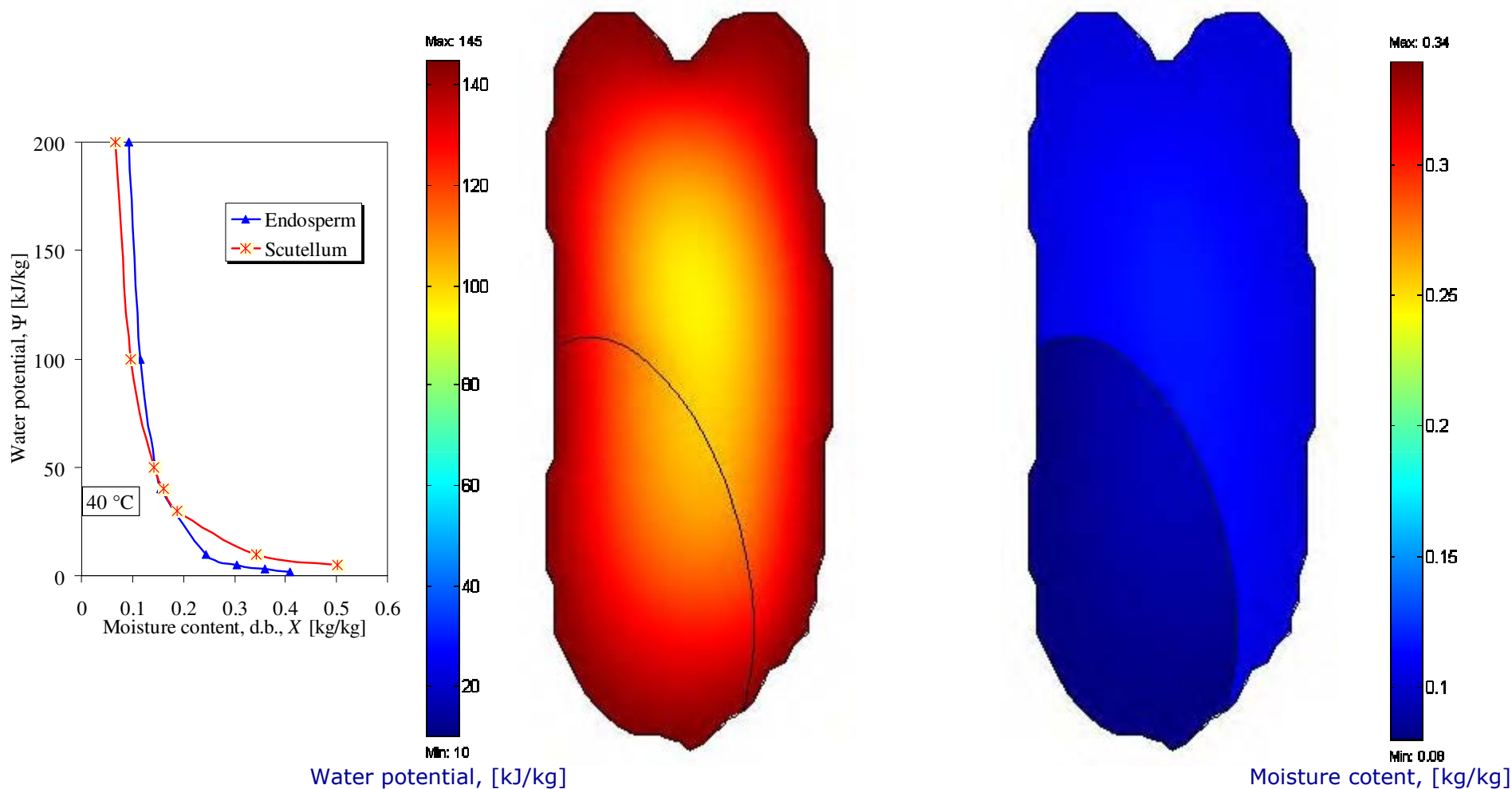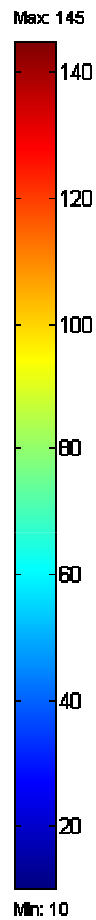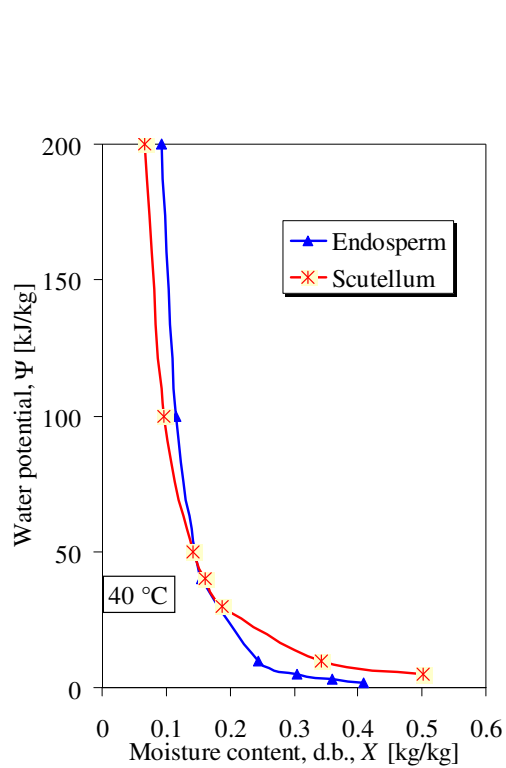
Modelled Ψ changes

X changes based on grad Ψ

Water potential, [kJ/kg]

Moisture cotent, [kg/kg]

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Modelled **Ψ changes**

***X* changes** based on grad Ψ

Water potential, [kJ/kg]

Moisture cotent, [kg/kg]

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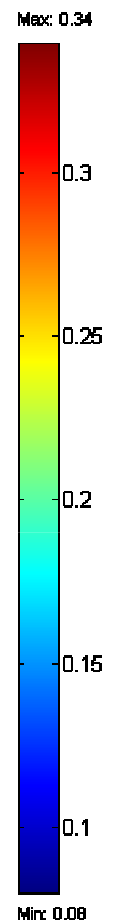**Institute of Biosystems Engineering**
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Ψ changes

X changes based on grad Ψ

Water potential, [kJ/kg]
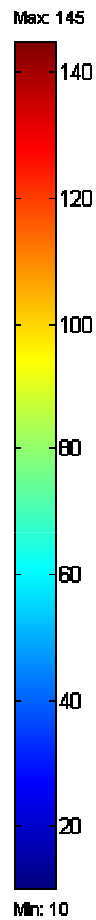
Moisture cotent, [kg/kg]

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Modelled Ψ changes

X changes based on grad Ψ

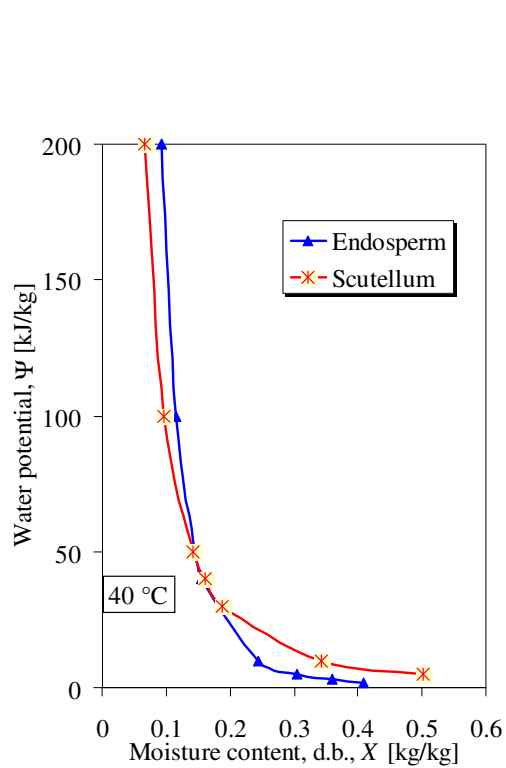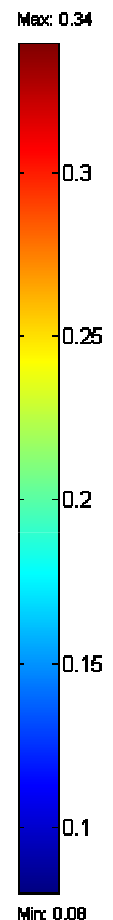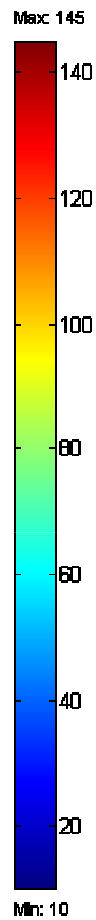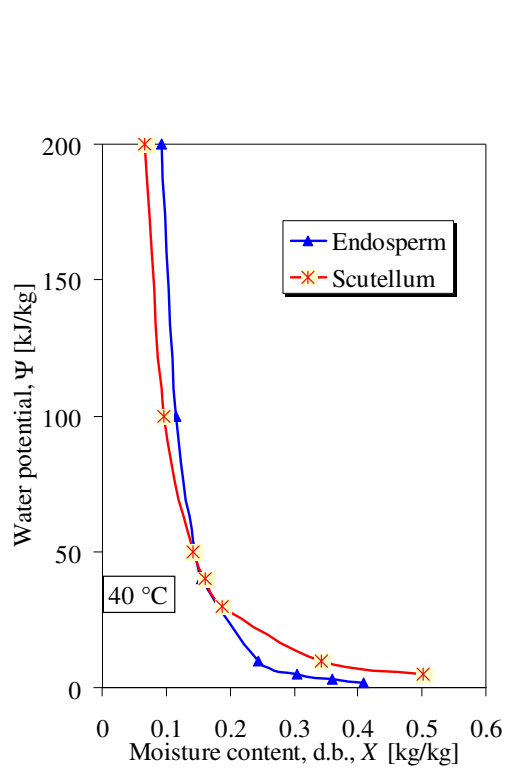

Water potential, [kJ/kg]

Moisture cotent, [kg/kg]

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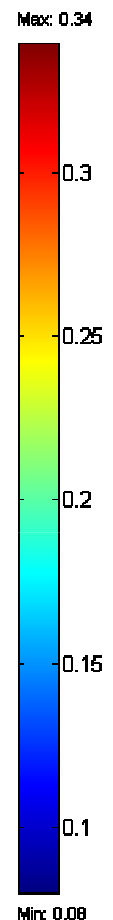**Institute of Biosystems Engineering**
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Modelled Ψ changes

X changes based on grad Ψ

Max: 145

140

120

100

80

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Min: 10

Water potential, [kJ/kg]

Max: 0.34

0.3

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Moisture cotent, [kg/kg]

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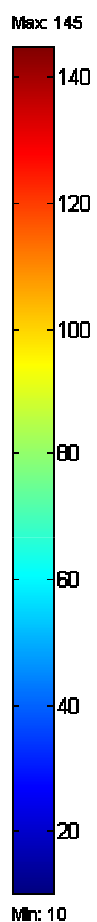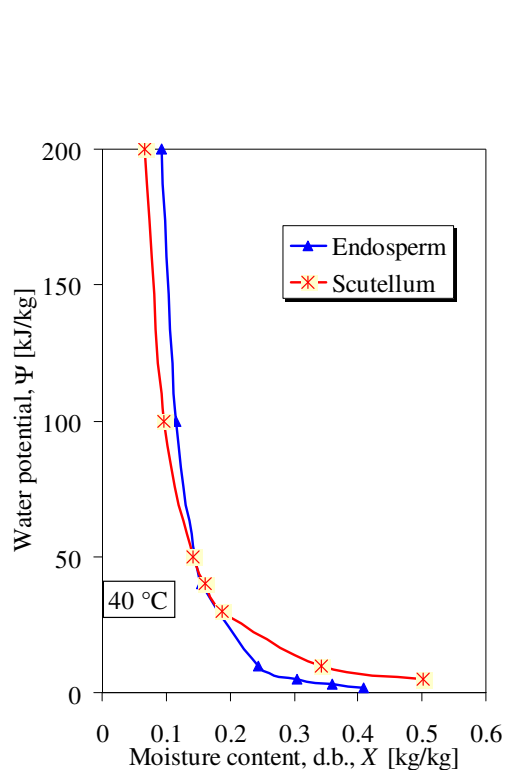
Moisture content, d.b., X [kg/kg]

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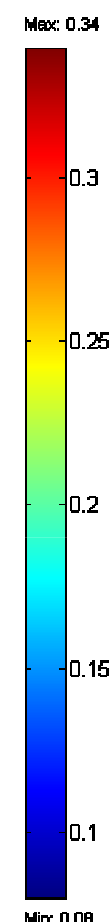**Institute of Biosystems Engineering**
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Modelled
Ψ changes

***X* changes** based on grad Ψ

Water potential, [kJ/kg]
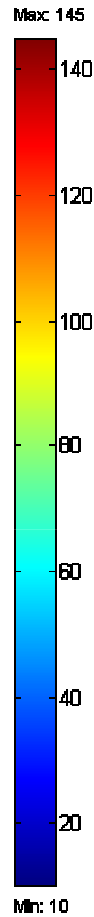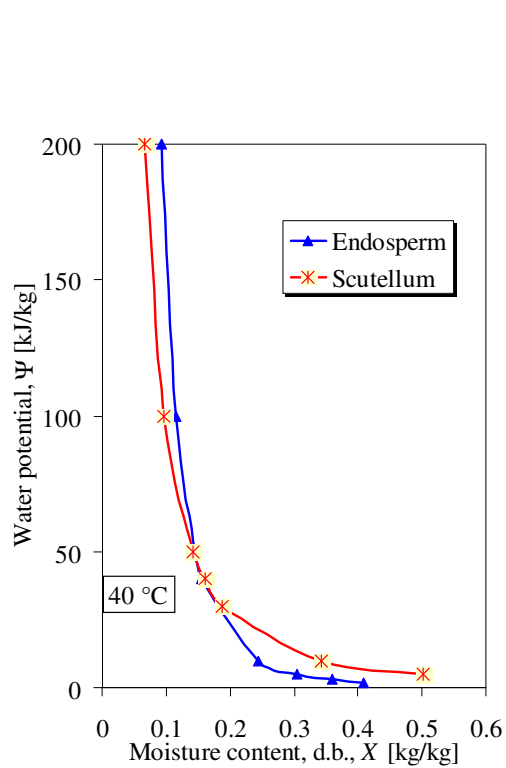
Moisture cotent, [kg/kg]

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Modelled **Ψ changes**

***X* changes** based on grad Ψ

Water potential, [kJ/kg]

Moisture cotent, [kg/kg]

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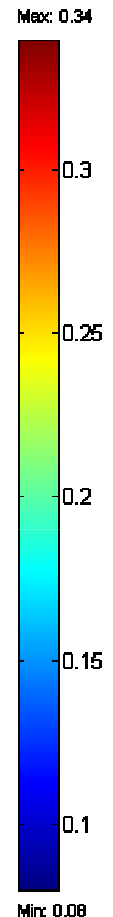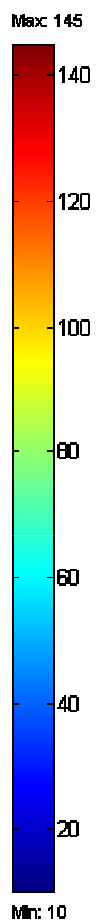**Institute of Biosystems Engineering**
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Modelled
Ψ changes

***X* changes** based on grad Ψ

Water potential, [kJ/kg]
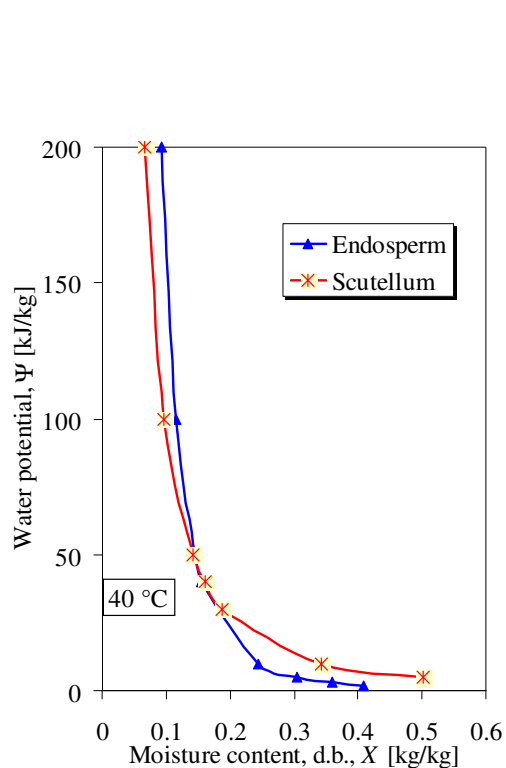
Moisture cotent, [kg/kg]

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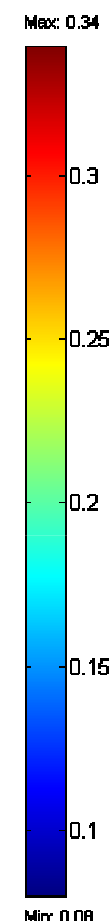**Institute of Biosystems Engineering**
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Modelled
Ψ changes

X changes based on grad Ψ
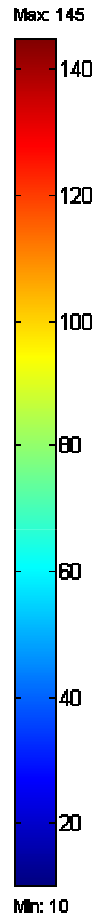

Water potential, [kJ/kg]
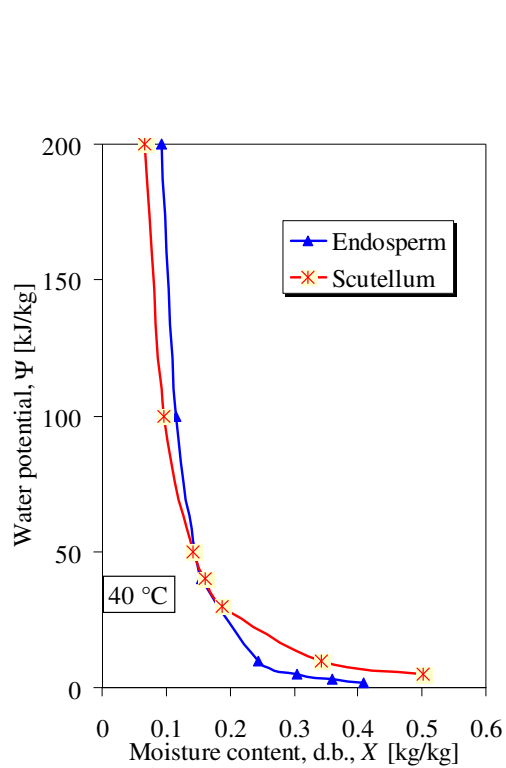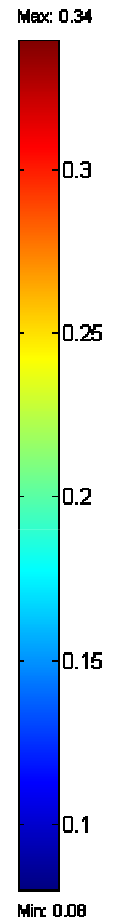
Moisture cotent, [kg/kg]

Modelled Ψ changes

X changes based on grad Ψ

Water potential, [kJ/kg]

Moisture cotent, [kg/kg]

CONCLUSIONS

- In the governing equation using water potential as the driving force has exact physical meaning, instead of the moisture gradient drying force.

- One multiphysical equation system gives more accurate solutions instead of iteration solutions.

- Further studies needs to be done to determine water potential above hygroscopic moisture content.

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THANK YOU FOR YOUR ATTENTION!

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