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Inverse Estimation of the Flow Resistivity Tensor of Open-Cell Foams from Experimental Data and Darcy's Flow Simulations

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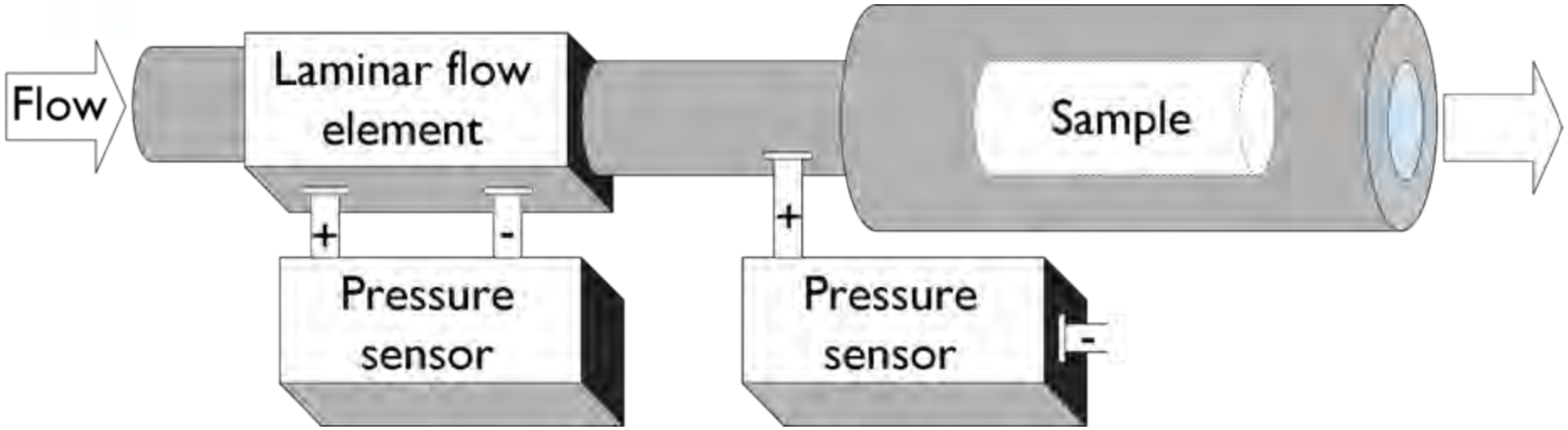
Objectives

Method Göransson and Guastavino → Experimental set up; Inverse estimation

This work → Extend Verification; Improvements; Application

Standard flow resistivity measurement (ISO 9053)

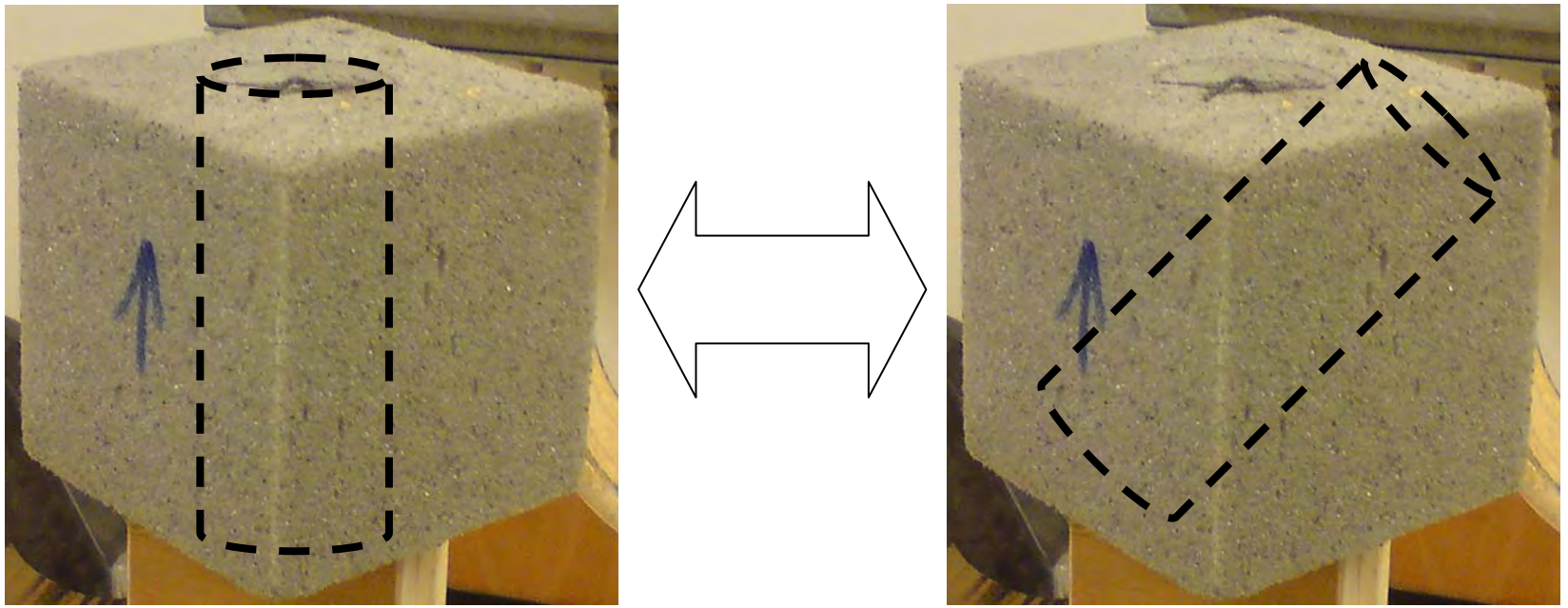


Limited repeatability

Unidirectional measurement

$$\sigma^{cyl} = \frac{\Delta p}{L \cdot V}$$

Standard flow resistivity measurement (ISO 9053)



Full 3D anisotropic flow resistivity tensor

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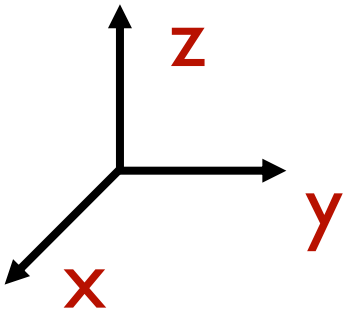
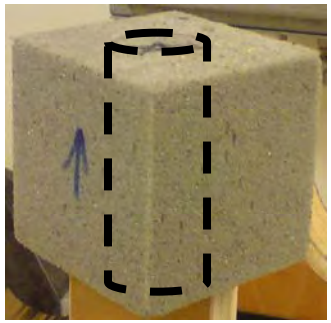
Flow resistivity tensor

$$\boldsymbol{\sigma} = \begin{bmatrix} \boldsymbol{\sigma}_{xx} & \boldsymbol{\sigma}_{xy} & \boldsymbol{\sigma}_{xz} \\ & \boldsymbol{\sigma}_{yy} & \boldsymbol{\sigma}_{yz} \\ sym & & \boldsymbol{\sigma}_{zz} \end{bmatrix}$$





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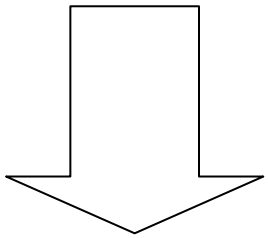
Method of Göransson and Guastavino

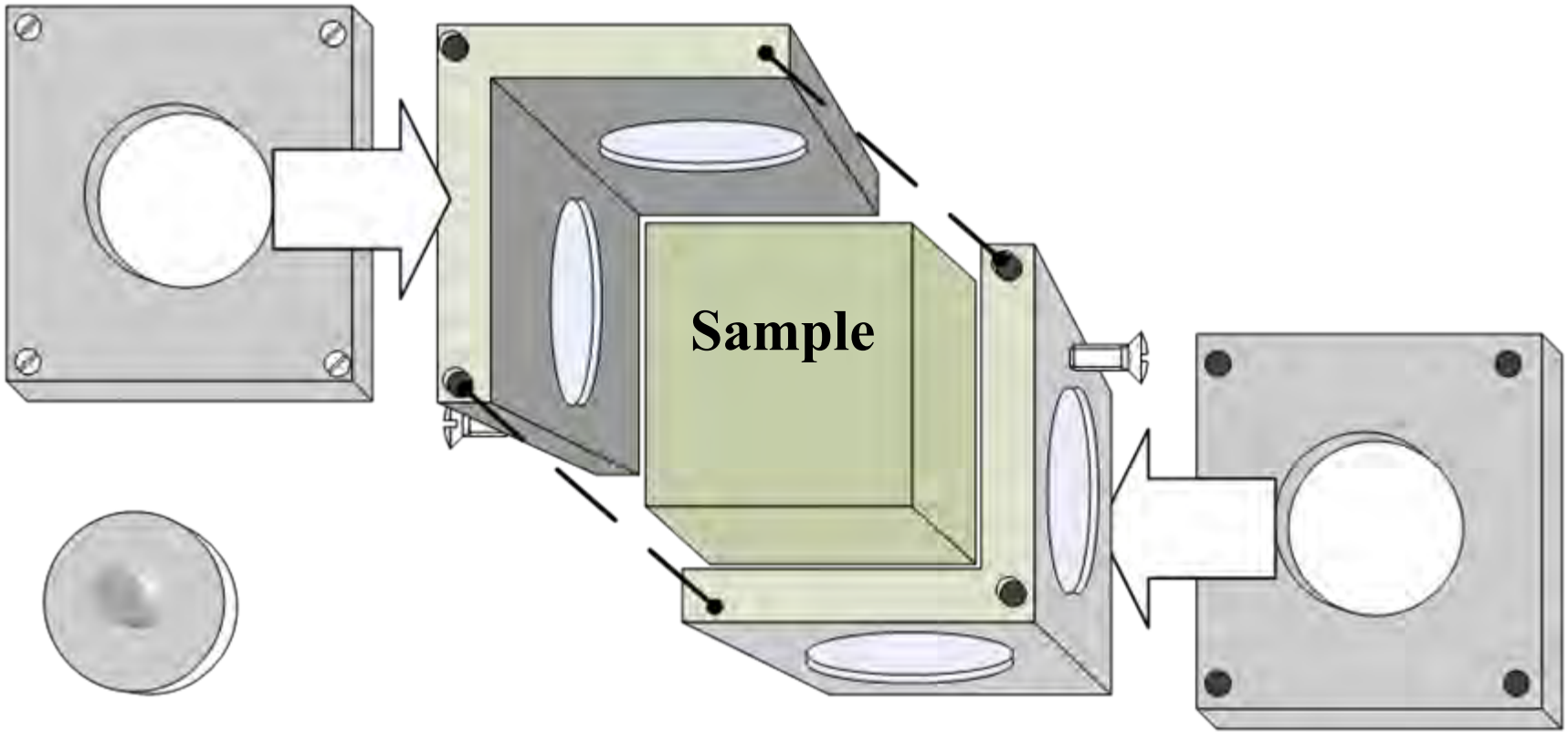
Identification of
full anisotropic flow resistivity tensor



Experimental set up

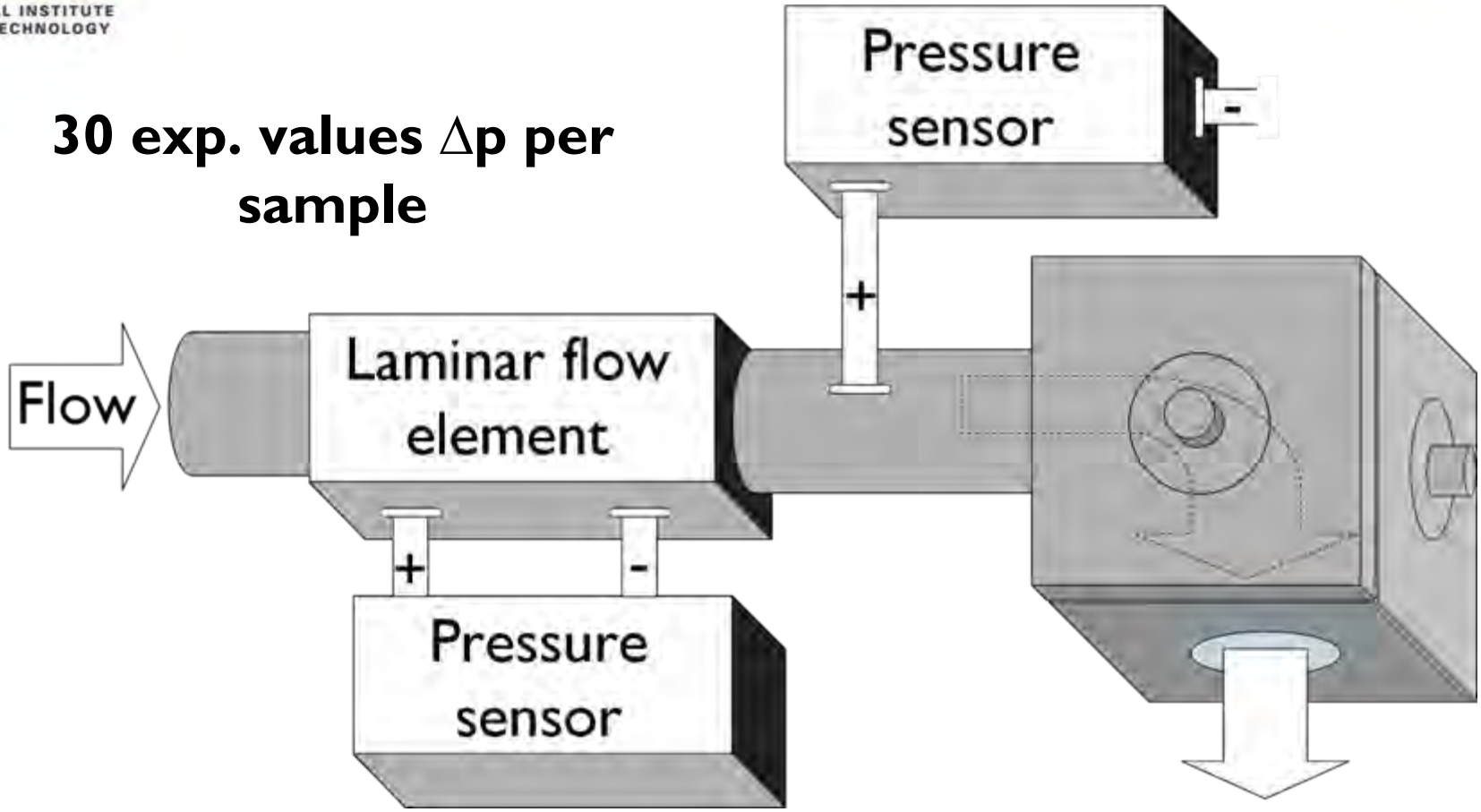
Inverse estimation with
COMSOL Earth Science Module

Experimental set up



Experimental set up

30 exp. values Δp per sample



Method of Göransson and Guastavino

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Experimental set up

Inverse estimation with COMSOL Earth Science Module

1. Governing Equation + Boundary Conditions
2. Modelling and solving with COMSOL
3. Optimisation loop

Governing Equation

Darcy's law

$$\boldsymbol{\sigma}^{-1} \cdot \nabla p = \mathbf{v}$$

+

Incompressibility condition

$$\nabla \mathbf{v} = 0$$

$$\nabla\left(\boldsymbol{\sigma}^{-1} \cdot \nabla p\right) = 0$$

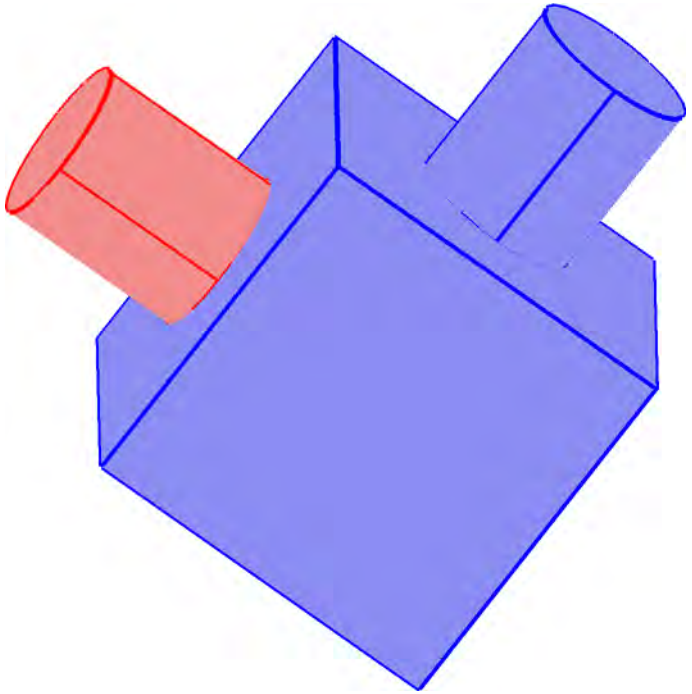
Solve for flow resistivity tensor **σ**

positive definite and symmetric

Boundary Conditions

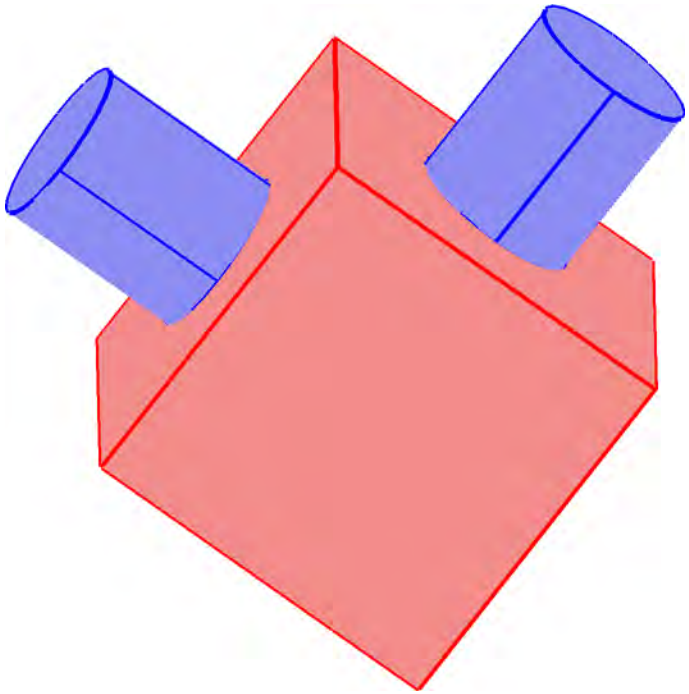
Inlet:

$$\nabla p \cdot \mathbf{n}_i = \overline{\sigma}\ \overline{\mathbf{v}} \cdot \mathbf{n}_i$$



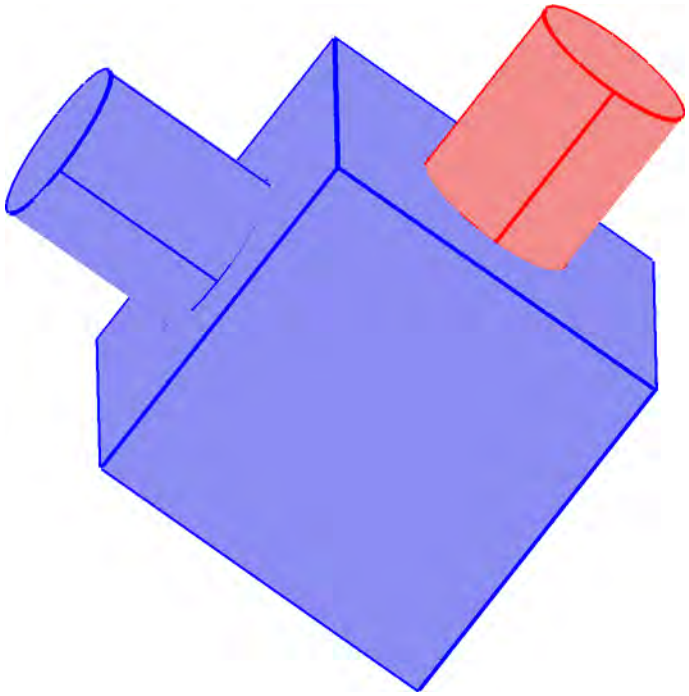
Boundary Conditions

Rigid walls: $\nabla p \cdot \mathbf{n} = 0$

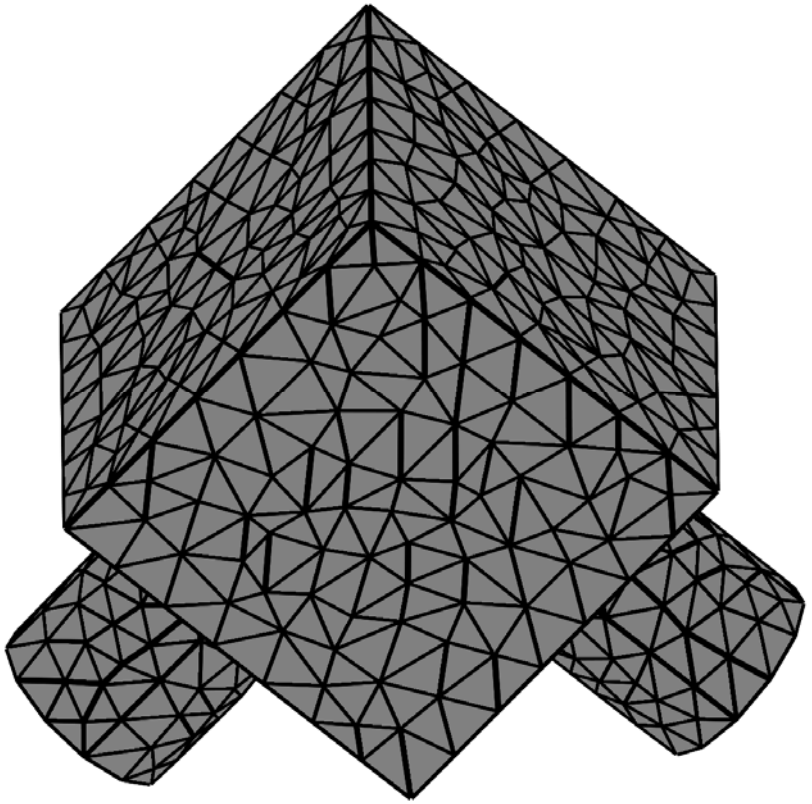


Boundary Conditions

Outlet: $p = 0$



Modelling with COMSOL

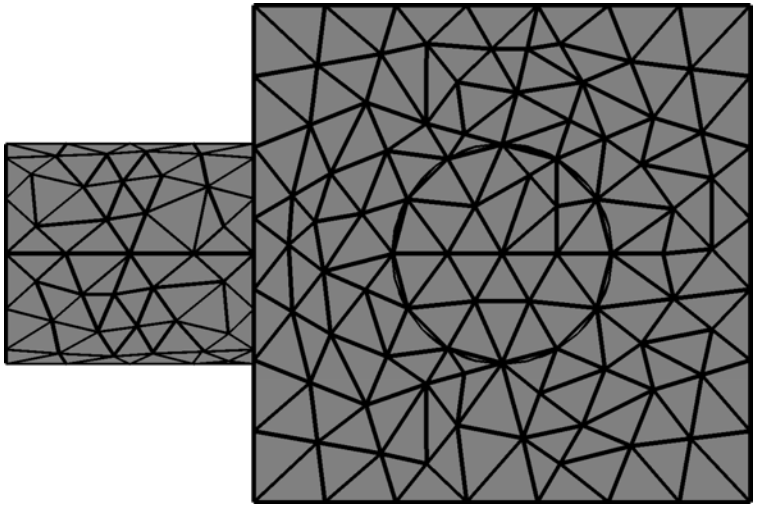


COMSOL Earth Science Module

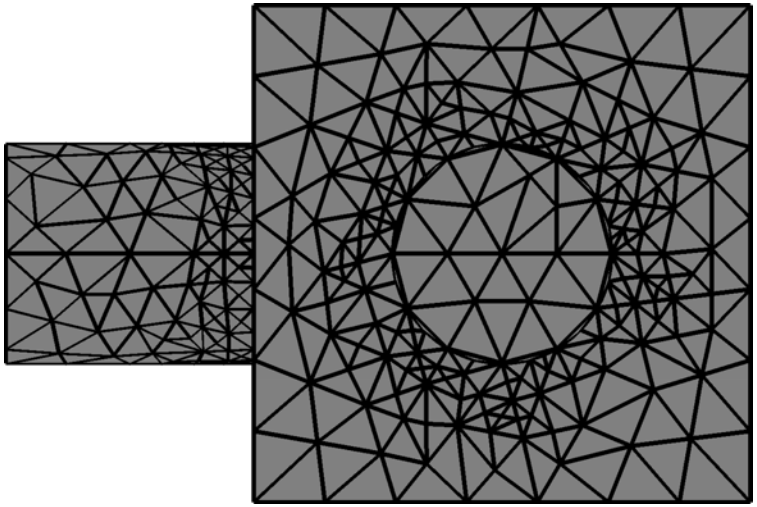
20000 DOF

Quadratic Lagrange polynomial tetrahedral elements

Mesh



Start with normal mesh



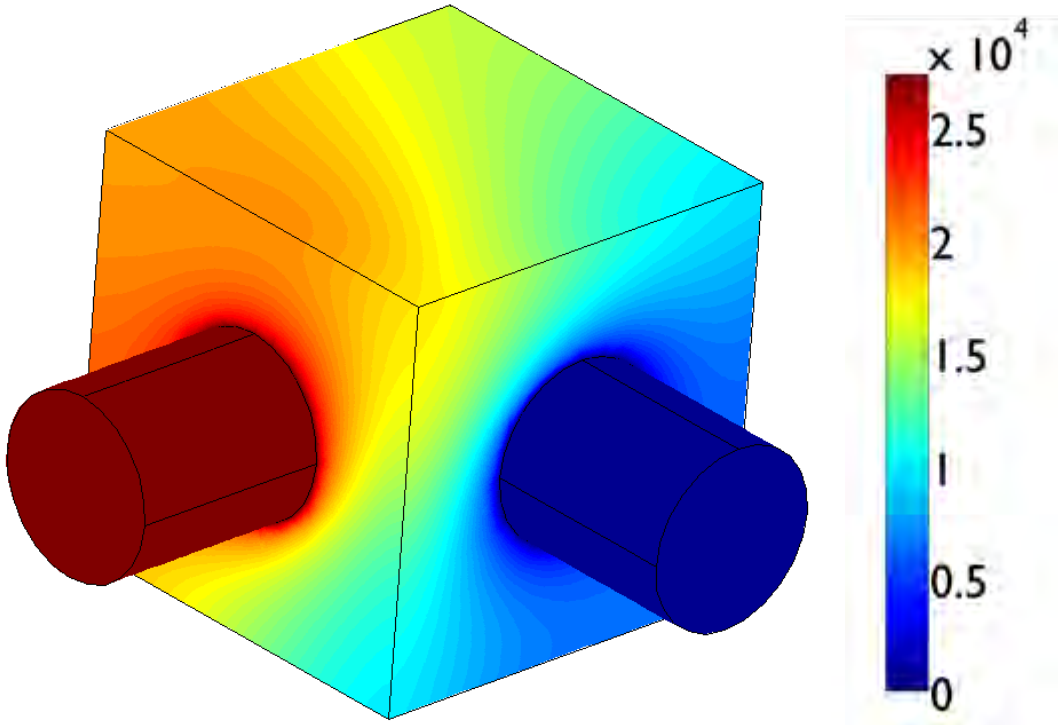
I adaptive mesh while solving

Solving with COMSOL

Solve for inlet pressure

Calculation time 1 solution: ± 7 seconds

Pressure distribution inside sample (Pa)



Optimisation loop

Using Svanbergs ”(Globally Convergent) Method of Moving Asymptotes: (GC)MMA”

Find $\boldsymbol{\sigma}$ so that
$||\Delta p_{meas} - \Delta p_{COMSOL}||$ is minimal

Constraint: $\boldsymbol{\sigma}$ positive definite

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Extended verification of estimation procedure

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Case 1

$$\begin{bmatrix} 10000 & 3800 & 2800 \\ 3800 & 9800 & 5200 \\ 2800 & 5200 & 12400 \end{bmatrix}$$

Case 2

$$\begin{bmatrix} 10000 & -3800 & 2800 \\ -3800 & 9800 & -5200 \\ 2800 & -5200 & 12400 \end{bmatrix}$$

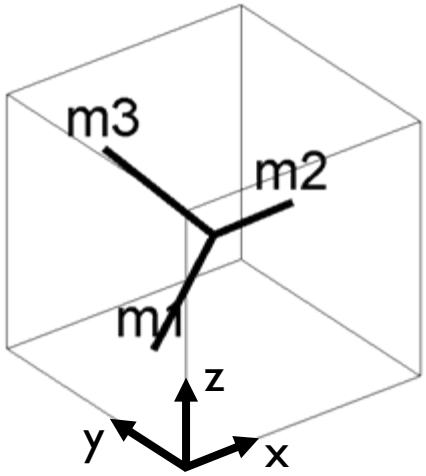
Case 3

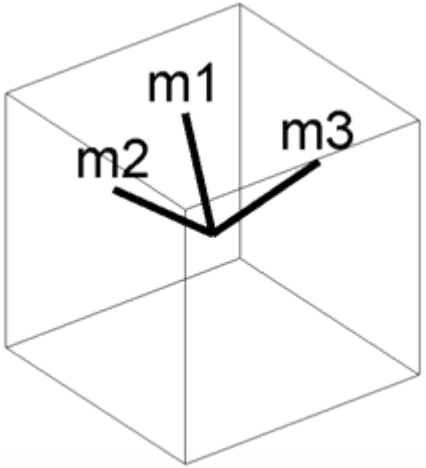
$$\begin{bmatrix} 10000 & 380 & 280 \\ 380 & 9800 & 520 \\ 280 & 520 & 12400 \end{bmatrix}$$

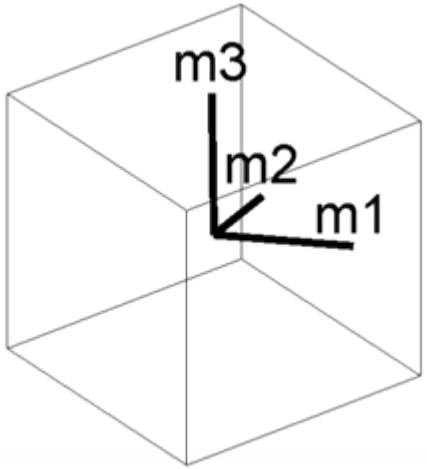
Case 4

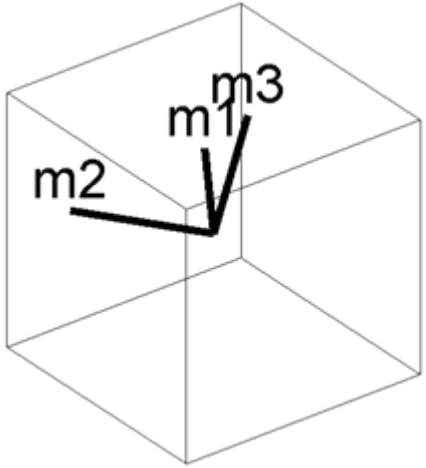
$$\begin{bmatrix} 10000 & -380 & 280 \\ -380 & 9800 & -520 \\ 280 & -520 & 12400 \end{bmatrix}$$

[Pa s/m²]









Extended verification of estimation procedure

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Largest relative error in flow resistivity

Case	GCMMA	MMA
Case 1	0.003 %	0.10 %
Case 2	0.002 %	0.11 %
Case 3	0.03 %	0.03 %
Case 4	0.52 %	0.02 %

Estimation procedure works

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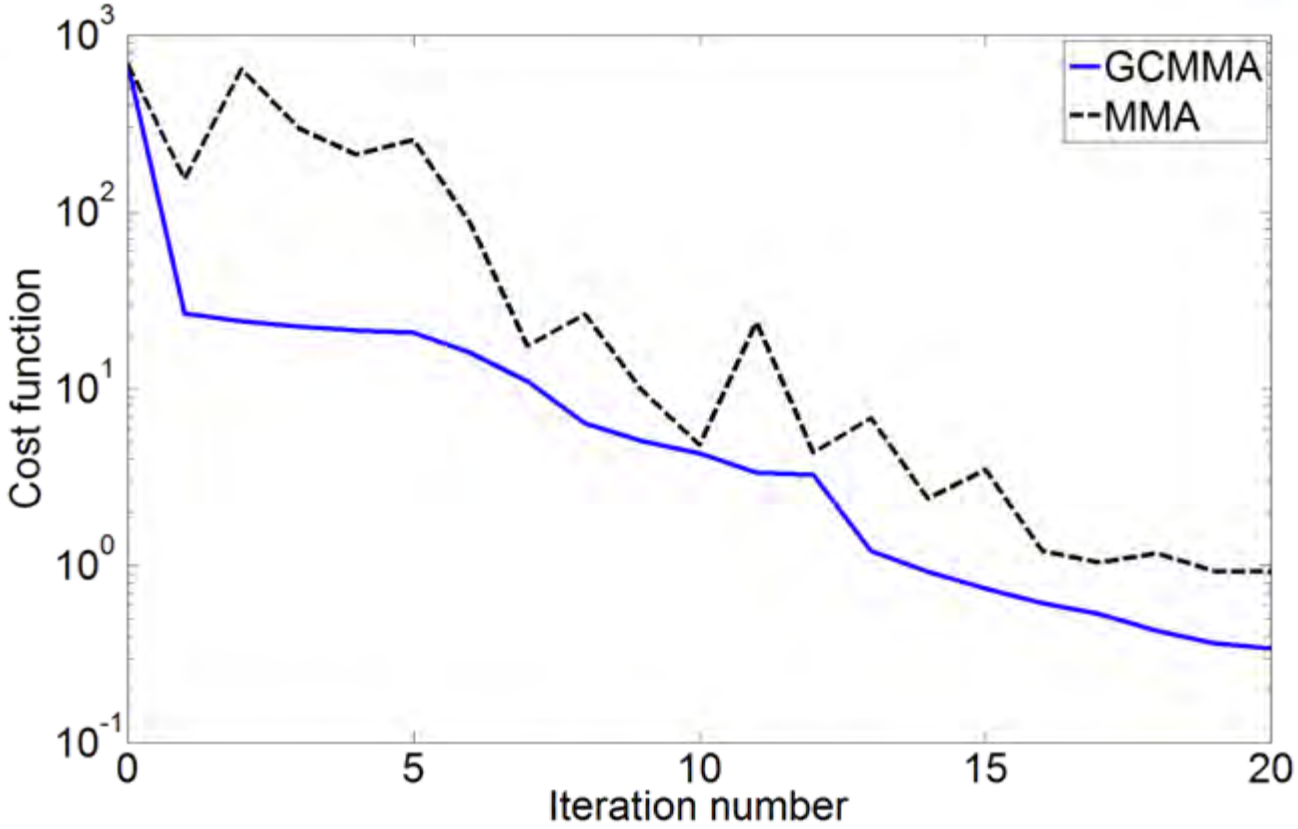
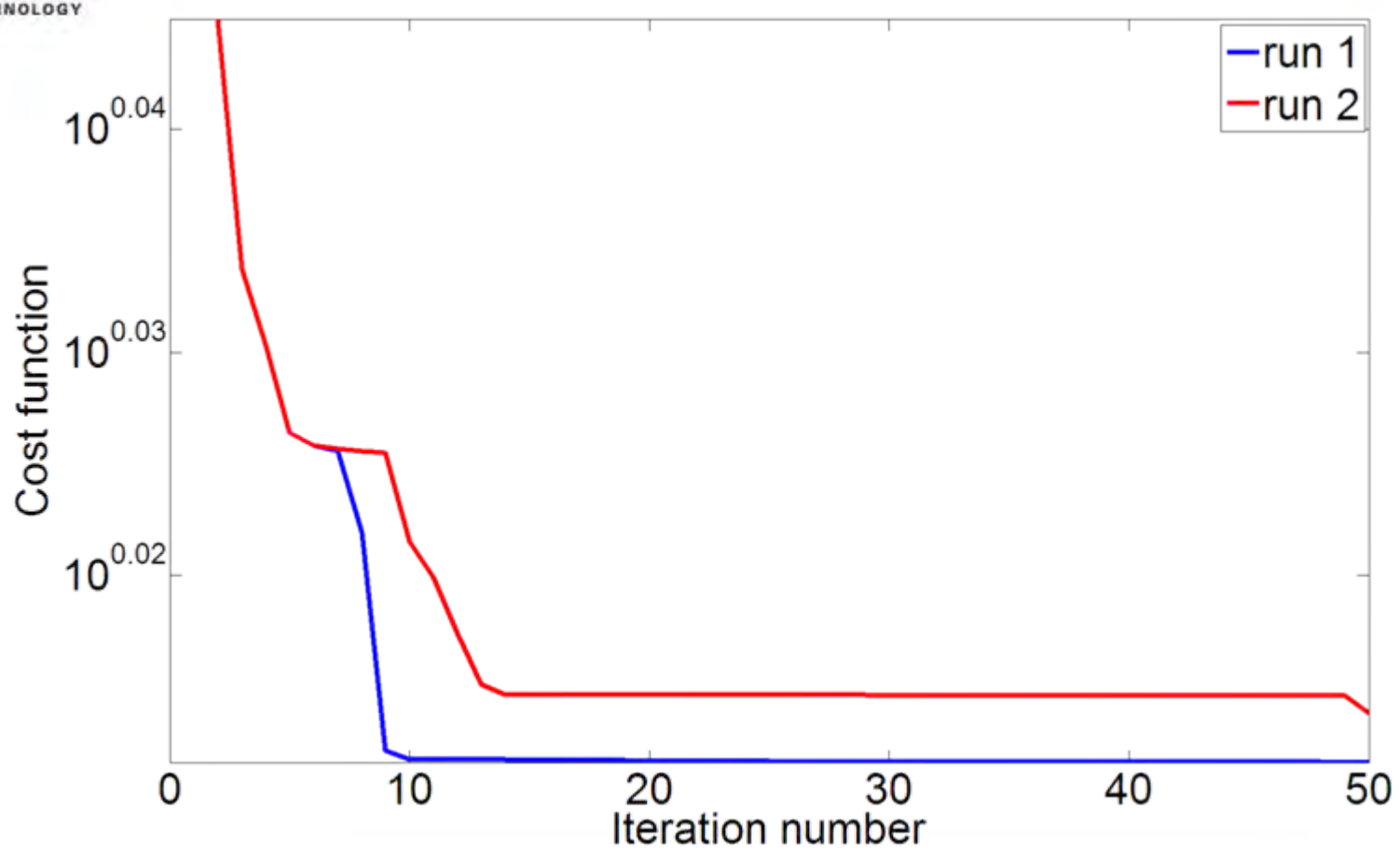
MMA vs. GCMMA

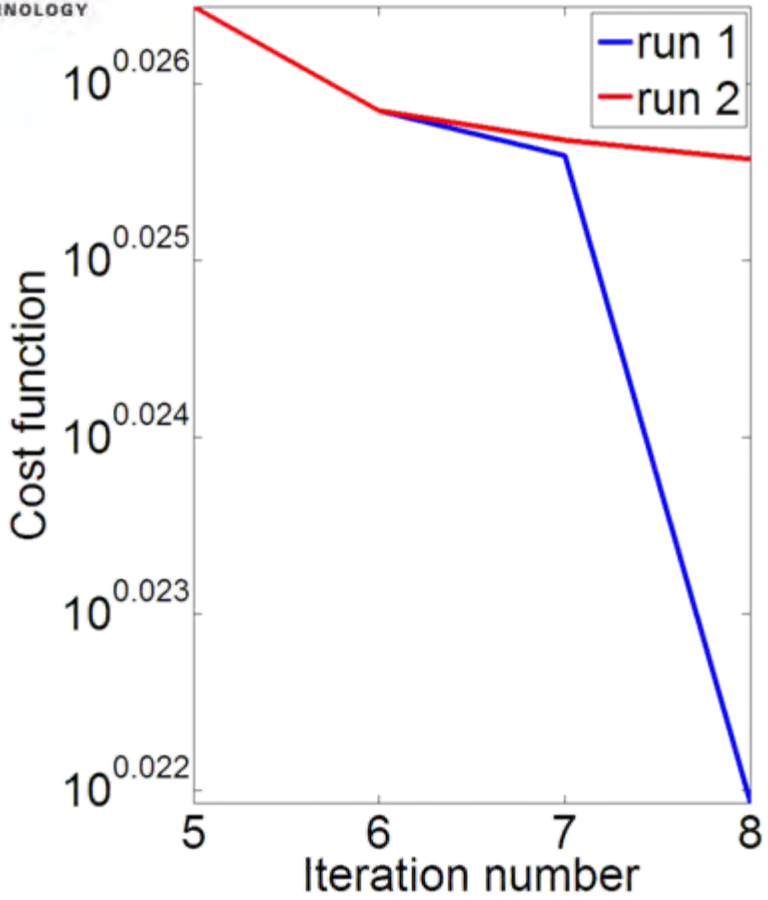


GCMMA is improvement

COMSOL observations



COMSOL observations



Adaptive mesh based on random algorithm

Different mesh possible

Different pressure output

COMSOL observations

Workaround: limit COMSOL output to 7 significant digits

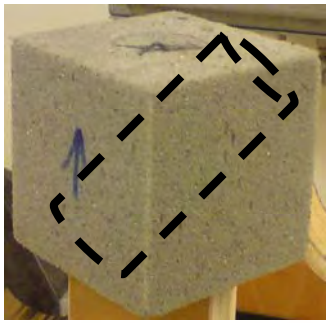
Suggestions?

Application to Melamine sample

Full anisotropic flow resisitivity tensor

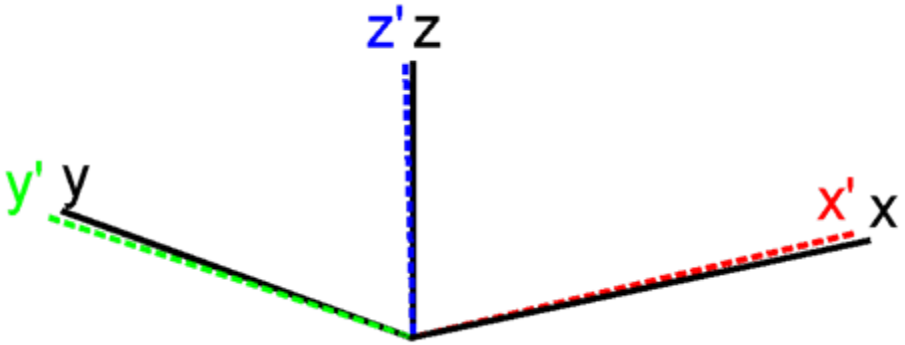
$$\boldsymbol{\sigma} = \begin{bmatrix} 9800 & 3 & -11 \\ 3 & 9700 & 8 \\ -11 & 8 & 10900 \end{bmatrix} \quad [\text{Pa s/m}^2]$$

Anisotropic $\boldsymbol{\sigma}$ dependent on sample orientation and manufacturing process



Application to Melamine sample

$$\begin{bmatrix} \boldsymbol{\sigma}_{x'} = 9800 \\ \boldsymbol{\sigma}_{y'} = 9700 \\ \boldsymbol{\sigma}_{z'} = 10900 \end{bmatrix}$$



x, y, z: geometric coord. system

x', y', z': material coord. system

Close to tranverse isotropy

Conclusions

Extend Verficiation

Method verified for wider range of applications

Improvements

*GCMMA optimiser
*Minimisation of effect adaptive meshing

Application

Melamine sample close to tranverse isotropic

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