

FEM Based Modeling in COMSOL Multiphysics and Design of Control of Distributed Parameter Systems

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Abstract: This paper presents a finite element method based modeling and design of control for distributed parameter systems. First, models of distributed parameter systems in the form of lumped-input/distributed-output systems and structure of control loop are introduced. Next, modeling of temperature fields of the casting die as distributed parameter systems in preheating process is performed in COMSOL Multiphysics Environment. Distributed characteristics of temperature fields either in stationary or transient form obtained there by numerical solution are exported to the MATLAB & Simulink Environment, where lumped and distributed models for control synthesis purpose are created. Finally, by means of Distributed Parameter Systems Blockset for MATLAB & Simulink - Third-Party Product of The MathWorks, feedback distributed parameter system of control is arranged. There for chosen control synthesis method, simulation of control process of temperature fields in the casting die is executed and analyzed.

Keywords: distributed parameter systems, partial differential equations, modeling, control, numerical methods.

1. Introduction

Dynamical systems analyzed in the engineering practice are very frequently distributed parameter systems (DPS) defined on complex-shape 3D definition domains. In systems and control theory, DPS are mathematically described by partial differential equations (PDE) with given boundary and initial condition, [1], [2], [3].

Temperature fields of the die in the casting process also belong to the class of DPS. There in order to obtain the desired structure during solidification, the process requires a specific temperature field of the die. Modeling, simulation and evaluation of real-time experiments in this area is now widely accepted

as an important tool in product design and process development to improve productivity and casting quality.

For analysis and control synthesis purposes, a benchmark of the casting processes with two-part steel die of complex-shape with embedded electric heating elements, water-cooled copper chills and thermocouples was designed. It has to be able to embrace of typical effects and critical moments of casting process. Required temperature field is achieved by means of actuating quantities – heating elements, which are suitable displaced in 5 zones inside the casting die.

Temperature fields of the casting die for zone located preheating process in this benchmark were modeled and studied by using of COMSOL Multiphysics, which is a finite elements method (FEM) software product package. Based on FEM results of modeling, numerical models in the form of a lumped-input/distributed-output system (LDS) were created in Matlab. These models serve for distributed parameter control synthesis purpose.

Analysis of models, design of control synthesis, but also simulation of temperature fields was carried out using the Distributed Parameter Systems Blockset for MATLAB & Simulink - Third-party software product of The MathWorks, developed at the Institute of Automation, Measurement and Applied Informatics, Faculty of Mechanical Engineering, Slovak University of Technology [8].

2. LDS representation of DPS

Generally, DPS are systems whose state/output variables, $X(x,y,z,t)/Y(x,y,z,t)$ are distributed variables or fields of variables, where (x,y,z) are spatial coordinates in 3D. As to the input/output relations, PDE define distributed-input/distributed-output systems (DDS) between distributed input, $U(x,y,z,t)$ and distributed output variables, $Y(x,y,z,t)$, at initial and boundary conditions given. Distributed

parameter systems very frequently are found in various technical and non-technical branches in the form of LDS, [5], [6], see Fig. 1.

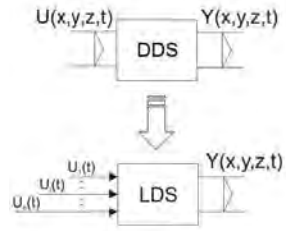


Figure 1. Lumped-input/distributed-output system representation of distributed parameter systems.

Output of the linear LDS in time domain, or in s-domain is in the form:

$$Y(\bar{x}, t) = \sum_{i=1}^n Y_i(\bar{x}, t) = \sum_{i=1}^n \mathcal{G}_i(\bar{x}, t) \otimes U_i(t) \quad (1)$$

$$Y(\bar{x}, s) = \sum_{i=1}^n Y_i(\bar{x}, s) = \sum_{i=1}^n S_i(\bar{x}, s) U_i(s) \quad (2)$$

where $\bar{x} = (x, y, z)$ is position vector in 3D, $U_i(t)$ - lumped input quantity, $\mathcal{G}_i(\bar{x}, t)$ - i -th distributed impulse response, \otimes - denotes convolution product, $Y_i(\bar{x}, t)$ - system response to the i -th input, $S_i(\bar{x}, s)$ - i -th transfer function, [6]. When $U_i(t)$ is a unit-step (Heaviside) function, $Y_i(\bar{x}, t)$ is in the form of distributed transient response function $\mathcal{H}_i(\bar{x}, t)$.

For a discrete-time system considering zero-order hold (ZOH) units, the overall distributed output variable of LDS and ZOH can be expressed in the form:

$$Y(\bar{x}, k) = \sum_{i=1}^n Y_i(\bar{x}, k) = \sum_{i=1}^n \mathcal{G}H_i(\bar{x}, k) \oplus U_i(k) \quad (3)$$

where \oplus denotes convolution sum. For points $\bar{x}_i = (x_i, y_i, z_i)$ located in surroundings of lumped input variables, $U_i(t)$, where partial distributed transient responses $\mathcal{H}_i(\bar{x}_i, t)$ attain maximal amplitudes, partial particular distributed output variables are obtained in time-domain, or in z-domain respectively:

$$\{Y_i(\bar{x}_i, k) = \mathcal{G}H_i(\bar{x}_i, k) \oplus U_i(k)\}_{i=1, n} \quad (4)$$

$$\{Y_i(\bar{x}_i, z) = SH_i(\bar{x}_i, z) U_i(z)\}_{i=1, n} \quad (5)$$

For the space dependency and in steady-state we can define reduced transient step responses between i -th input variable at point $\bar{x}_i = (x_i, y_i, z_i)$ and corresponding partial particular distributed output variable in steady-state:

$$\left\{ \mathcal{H}R_i(\bar{x}, \infty) = \frac{\mathcal{H}H_i(\bar{x}, \infty)}{\mathcal{H}H_i(\bar{x}_i, \infty)} \right\}_{i=1, n} \quad (6)$$

Dynamics of LDS is decomposed to time and space components. In the time dependency, there are, discrete transfer functions of the form $\{SH_i(\bar{x}_i, z)\}_{i=1, n}$ and in the space direction there are $\{\mathcal{H}R_i(\bar{x}, \infty)\}_{i=1, n}$.

3. Feedback control loop based on LDS

Structure of DPS feedback control loop based on LDS is on Fig. 2, where the goal of control is to ensure the steady-state control error to be minimal:

$$\min \|\bar{E}(x, \infty)\| = \min \|W(\bar{x}, \infty) - Y(\bar{x}, \infty)\| \quad (7)$$

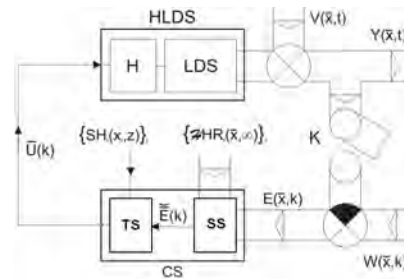


Figure 2. Distributed parameter feedback control loop: HLDS - LDS with zero-order holds $\{H_i\}_i$ on the input, CS - control synthesis, TS - control synthesis in time domain, SS - control synthesis in space domain, $Y(\bar{x}, t)$ - distributed controlled variable, $W(\bar{x}, k)$ - control variable, $V(\bar{x}, t)$ - disturbance variable, $E(\bar{x}, k)$ - control error, K - time/space sampling.

In the block SS, approximation of distributed control error $E(\bar{x}, k)$, on the set of reduced

steady-state distributed step responses $\{\mathcal{A}HR_i(\bar{x}, \infty)\}_i$, is solved.

$$\begin{aligned} \min_{\bar{E}_i} \left\| E(\bar{x}, k) - \sum_{i=1}^n E_i(k) \mathcal{A}HR_i(\bar{x}, \infty) \right\| &= \\ = \left\| E(\bar{x}, k) - \sum_{i=1}^n \bar{E}_i(k) \mathcal{A}HR_i(\bar{x}, \infty) \right\| & \quad (8) \end{aligned}$$

As the solution of approximation problem, the control errors vector $\bar{\bar{E}}(k) = \{\bar{E}_i(k)\}_i$ enters into the block TS, where vector of control variables, $\bar{U}(k)$, is generated by controllers, $\{R_i(z)\}_i$ in single-parameter control loops.

4. Benchmark casting plant

For study of the phenomena occurring during casting solidification, from a control point of view, control system development as well as mathematical model validation, a benchmark of the casting processes was designed, see Fig. 3.

The core item is the two-part steel die (mould) of a complex-shape mounted in the frame of the ejector mechanism. Inside of the casting die are built-in electric heating elements, water-cooled copper chills and thermocouples, see Fig. 4. Location of these elements has been carefully designed in order to have the possibility of preheating the die in 5 zones achieving desired temperature profile as well as directional solidification of the casting by means of active heat removal. The main heating circuit is divided into five independent circuits.

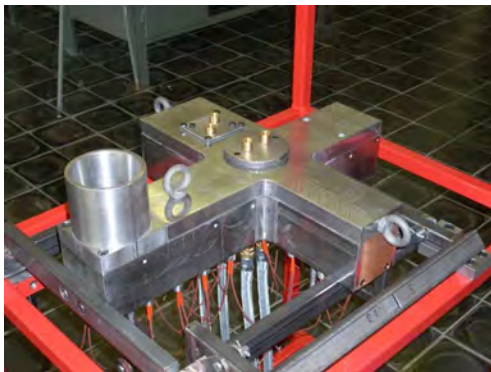


Figure 3. Benchmark casting plant.

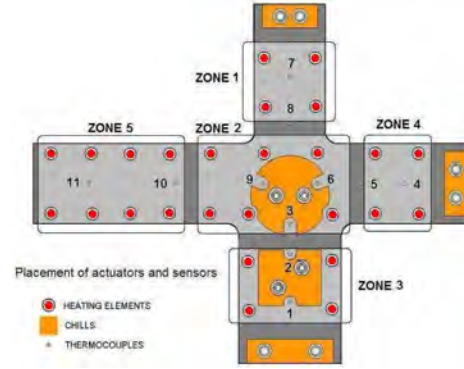


Figure 4. Bottom side of the steel casting die.

5. Modeling of temperature fields of casting die in COMSOL Multiphysics

The casting die is one of the key components of a casting. It is well known, that the quality of castings is affected strongly by the surface quality and the distribution of temperature in the die. Temperature field of the die in the casting process is a typical case of DPS. For modeling of temperature fields as DPS based on FEM, wide possibilities are offered by COMSOL Multiphysics Environment with Heat Transfer Module.

Distribution of temperatures $T(\bar{x}, t)$ in the casting die over the definition domain $\Omega \in E_3$ for lumped input variables $\{U_i(\bar{x}, t)\}_{i=1,5}$ in the form of heat sources Q_i [W/m³] is modeled by PDE of parabolic type with initial condition T_{init} and boundary conditions for heat flux:

$$\begin{aligned} \frac{\partial T(\bar{x}, t)}{\partial t} - a \nabla^2 T(\bar{x}, t) &= \sum_{i=1}^5 U_i(\bar{x}, t) \quad (9) \\ -n(-\lambda \nabla T) &= h(T_{ext} - T) \\ T(\bar{x}, 0) &= T_{init} \end{aligned}$$

where $a = \lambda / \rho c$ is temperature conductivity [m².s⁻¹], T_{ext} is external temperature and T_{init} is initial temperature.

3D geometry model of the die was drawn in a CAD environment and imported into COMSOL Multiphysics, where all parameters of PDE and other necessary conditions for

simulation and data processing were specified through graphical user interface (GUI), Fig. 5.

There by means of the toolbar menu in the *Physics* menu subdomains of the die with their parameters were defined. Parameters like thermal conductivity, density, heat capacity and heat source either for Iron or Copper were loaded from Library material. Heating elements in the cylinder shape, where heat sources as actuating quantities were defined, were grouped into 5 subdomains of the die, each contains 4 – 8 elements. Similarly were defined as groups copper chills, next bottom and upper part of the die. Also in the *Physics* menu Boundary conditions were defined.

In the *Mesh* menu mesh for the given geometry was analysed, e.g. initial mesh, various predefined mesh sizes, etc. The Mesh Statistics dialog box, available from the Mesh menu, contains statistical data for the current mesh.

In the *Solve* menu temperature fields of the die either in stationary, or transient form were computed by FEM. Actuating of heating elements was performed separately in each zone.

Obtained temperature fields were analyzed by means of various forms of plots and their parameters, including AVI animation with help of the *Postprocessing* menu. Several results are on Fig. 6 - Fig. 9.

Finally, in the *File* menu Postprocessing data in the form of nodes, elements, data were exported either as txt or m-files.

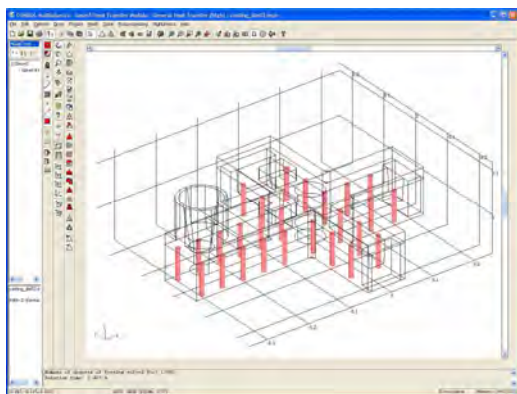


Figure 5. 3D geometry model of the casting die with location of heating elements (red).

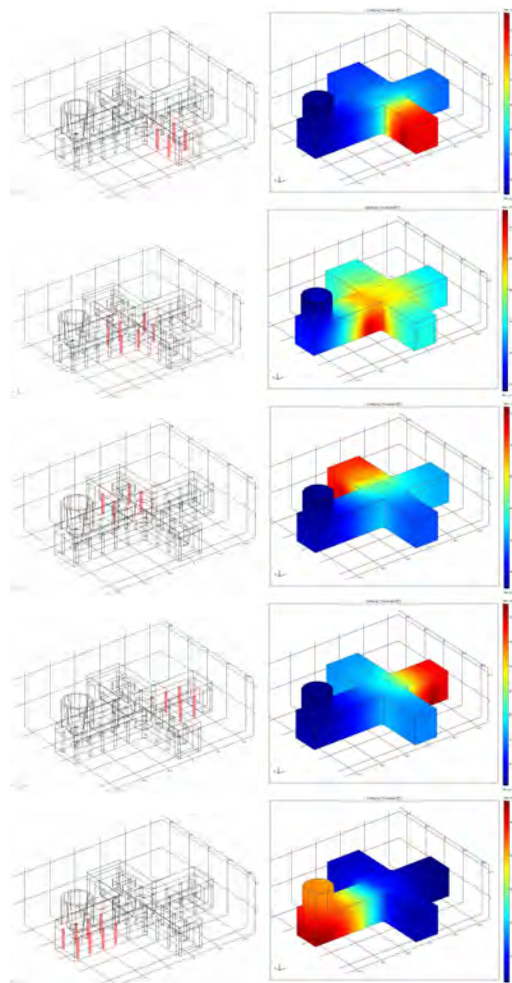


Figure 6. Temperature fields in steady-state actuated by heating elements in Zone #1 – Zone #5.

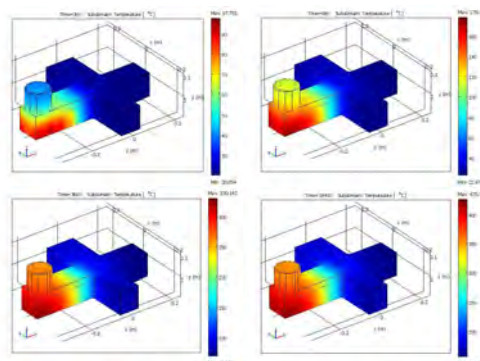


Figure 7. Temperature fields in transient form in given times, actuated by heating elements in Zone #5.

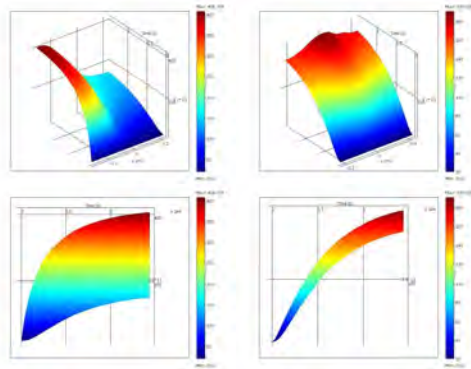


Figure 8. Temperature fields in transient form at $z=-0.05$ along x and y axis actuated by heating elements in Zone #5.

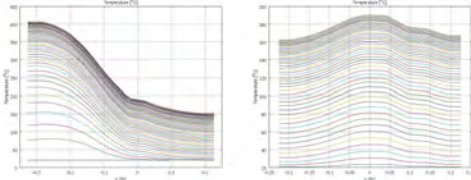


Figure 9. Temperature profiles in time $t = [0:300:14400]$ seconds actuated by heating elements in Zone #5.

6. DPS Blockset for MATLAB & Simulink

For a software support of modeling, control and design of Distributed Parameter Systems given on complex 3D domains of definition, DPS Blockset for MATLAB & Simulink as Third-Party MathWorks Product has been developed [8], see Fig. 10. The library of DPS Blockset shows Fig. 11.

The block *HLDS* models controlled distributed parameter systems as lumped-input/distributed-output systems with zero-order hold units. The *DPS Control Synthesis* provides feedback to distributed parameter controlled systems in control loops with blocks for PID, algebraic, state space and robust control. The block *DPS Input* generates distributed quantities which can be used as distributed control variables or distributed disturbances, etc. *DPS Display* presents distributed quantities with many options including export to AVI files. The block *DPS Space Synthesis* performs space synthesis as an approximation problem. The block *Tutorial* presents methodological

framework for formulation and solution of distributed parameter systems of control. The block *Show* contains motivation examples. The block *Demos* contains examples oriented to methodology of modeling and control synthesis. The *DPS Wizard* in step-by-step operation, by means of five model examples on 1D-3D with default parameters, gives a guide for arrangement and setting distributed parameter control loops.



Figure 10. DPS Blockset on the web portal of The MathWorks.

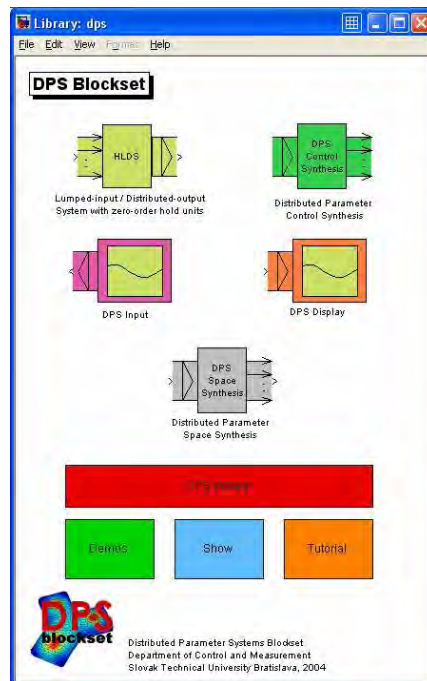


Figure 11. The library of DPS Blockset for MATLAB & Simulink.

7. Simulation of control for preheating of casting die

Temperature fields of the mould obtained by FEM solution in COMSOL Multiphysics were exported to Matlab-Simulink, in order to determine lumped and distributed models. Transfer functions $\{SH_i(\bar{x}_i, z)\}_{i=1,5}$ are used for control synthesis in time domain TS and reduced transient step responses in steady-state $\{\mathcal{H}R_i(\bar{x}, \infty)\}_{i=1,5}$ serves as basis functions for solution of approximation task in the control synthesis in space domain SS.

Transfer functions were identified in GUI *ident* of the System Identification Toolbox for points $\{\bar{x}_i = (x_i, y_i, z_i)\}_{i=1,5}$ in each zone, where temperatures attain maximal values, e.g. for actuating in Zone #5 result of identification is on Fig. 12 and obtained continuous and discrete transfer functions are in the form:

$$S_5(\bar{x}_5, s) = \frac{388,12(1940s + 1)}{(4180,4s + 1)(844,4s + 1)}$$

$$SH_5(\bar{x}_5, z) = \frac{24,26z - 22,81}{z^2 - 1,839z + 0.843}, \quad T = 120s$$

Simulation of the control process with five PID controllers was realised in the DPS Blockset for MATLAB & Simulink. By means of blocks of the DPS Blockset, DPS feedback control system with five PID controllers embedded in the DPS PID Synthesis block was arranged, see Fig. 13. Synthesis of PID controllers was adjusted in order to assure aperiodic running of the quadratic norm of distributed control error. Results of the PID control process for given reference quantity are on Fig. 14.

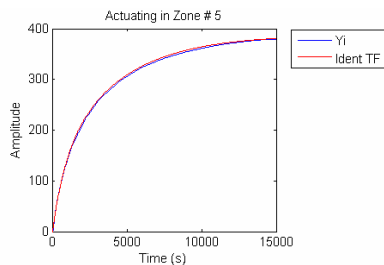


Figure 12. Identification of partial transient response in Zone #5.

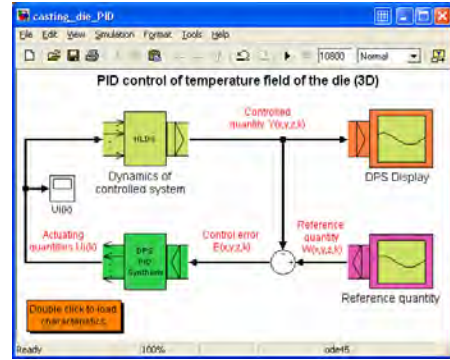


Figure 13. PID feedback control loop in DPS Blockset.

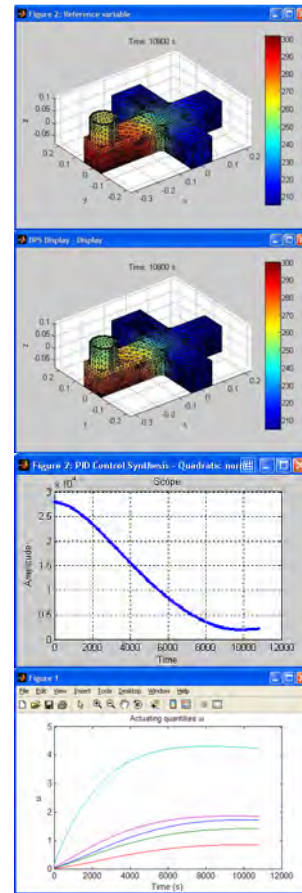


Figure 14. PID control of temperature field of the casting die: distributed reference quantity $W(\bar{x}, \infty)$, controlled quantity $Y(\bar{x}, \infty)$, quadratic norm of distributed control error $\|E(\bar{x}, k)\|$ and control quantities $U_i(k)$.

8. Conclusions

Nowadays, a number of software products, environments and virtual try-out spaces for numerical dynamical analysis of machines and processes are at disposal, practically in all engineering disciplines. Based on powerful FEM solvers and advanced specific options, COMSOL Multiphysics provides an efficient and accurate solution which users need. In the casting area it is the key solution to reduce manufacturing costs, shorten lead times for die developments and improve the casting process quality.

Results of numerical modeling of DPS are useful not only for study of their dynamics, but also for design of various control synthesis methods by using of DPS Blockset. Presented methodical approach of FEM based modeling and design of control of DPS is suitable for various technological processes.

9. References

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