Modeling of High Temperature Superconducting Tapes, Arrays and AC Cables Using COMSOL

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Abstract: In this paper we present a set of numerical models created with COMSOL Multiphysics. The set includes quantitative models of a coated conductor tape, an array of such tapes and a high temperature superconducting cable. Similar models were created in the past. An advantage of our approach is in additional transparency and traceability for a user, these are provided on one hand with existing COMSOL example and on the other hand via step by step validation of each model with trusted experimental and theoretical data.

Keywords: High temperature superconducting tapes, arrays and cables, ac losses, non-linearity.

1. Introduction

High temperature superconducting tapes of the second generation (coated YBCO tapes) and devices using the tapes show a trend to penetrate markets. Steps towards commercialization are made as the price of YBCO tapes goes down and other obstacles are resolved. One of the remaining technical obstacles is reduction of the losses, especially for AC applications such as cables. Numerical simulation using FEM becomes a proven tool to calculate AC losses in HTS tapes. The simulation is not trivial as the tapes show non linear resistivity, large aspect ratio and models of HTS tapes are generally not part of a standard FEM package.

Building models from scratch requires time and involves risks. The major risk is the typo. When creating complex models, typos can arise. It is therefore good practice to use validated paths. The COMSOL Multiphysics modeling environment contains a number of predefined models that provide such path for further manipulation.

2. Governing equations

Several formulations commonly used to solve Maxwell equations with 2D numerical models of superconductors are listed in [1, 2]. To illustrate our approach, we use here the $\vec{H}$-field formulation, see Eq. (1):

$$\mu\partial\vec{H}/\partial t + \nabla\times\rho\nabla\times\vec{H} = 0; \qquad (1)$$
$$\vec{J} = \nabla\times\vec{H}; \rho = \rho(J).$$

The formulation is well documented, other advantages of this formulation (such as use of the edge elements) are explained in [1]. Obviously, other formulations can be treated the same way.

For COMSOL modeling Eq. (1) presents a challenge as it involves cross products, which are not yet present in the general form PDE of COMSOL. They can however be implemented through the weak form and using existing example of COMSOL as explained in the next sections.

3. Method

For our purpose (to model a superconductor cross-section with minimum changes in COMSOL) an example exists in COMSOL: AC/DC module; Quasi-Statics, Magnetic; in-plane induction currents – vector potential. In this example Eq. (2) is the driving equation:

$$\sigma d\partial\vec{A}/\partial t + \nabla\times d(\mu_0^{-1}\mu_r^{-1}\nabla\times\vec{A}) = d\vec{J}^e \qquad (2)$$
$$\vec{B} = \nabla\times\vec{A};\ \sigma = \sigma(\vec{E});\ \vec{E} = -\partial\vec{A}/\partial t - \nabla\Phi .$$

One can solve Eq. (1), by solving Eq. (2) with a proper substitution of the variables and letting $d\equiv1$ and $\vec{J}^e\equiv0$. Tables 1, 2 provide the needed substitution for the $\vec{H}$-field formulation.

Table 1: Substitution of variables for $\vec{H}$-formulation

Original description	Para-meter	Substitute for HTS	Para-meter
Magnetic vector potential	A_x A_y	Magnetic field	H_x H_y
Conductivity	σ=1/ρ	permeability	$\mu_0*\mu_r$
Relative permeability	μ_r	conductivity	1/ρ
Absolute permeability	μ_0		1

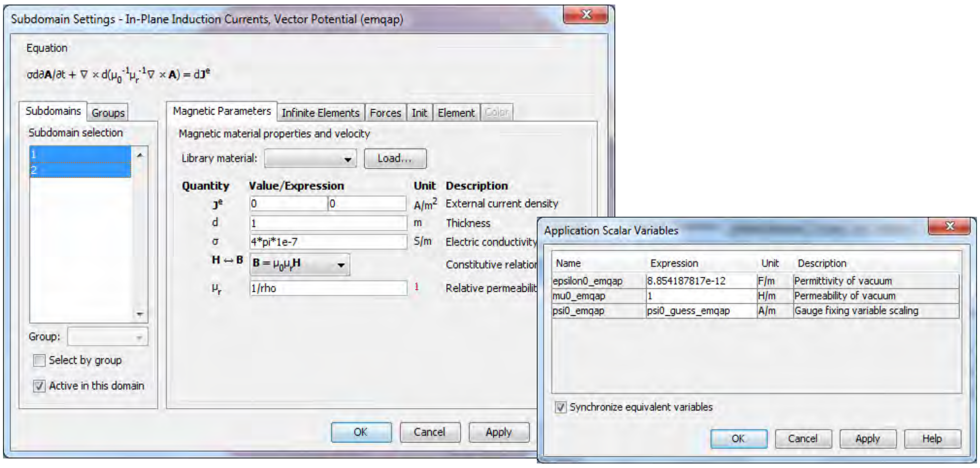

Figure 1. Sub-domain setting of the parameters in GUI of COMSOL Multiphysics 3.5a.

In COMSOL Multiphysics 3.5a GUI the substitutions appear as shown in Fig. 1. In the background, COMSOL computes a number of useful variables, these can be found in the Equation System. Two most important of them are listed in Table 2.

Table 2: Two useful variables computed by COMSOL

Parameter	HTS substitute	Parameter
Curl A_{z_emqap}	-	Curl H_z
B_{z_emqap}	Current density	J_z

The next step is to actually model a HTS tape. Resistivity ρ of a HTS tape is described by the so-called power law, see Eq. (3).

$$\rho(J_z) = \frac{E_c}{J_c}\left|\frac{J_z}{J_c}\right|^n . \qquad (3)$$

When modeling a superconductor placed in a non-conducting environment, the high gradients can cause non-convergence. This problem is solved by setting an ambient resistivity of several orders higher than that of the HTS tape. Similarly, the resistivity of HTS is kept above 10^{-18} Ωm. Therefore Eq. (3) is implemented into the COMSOL example using Eq. (4).

$$Rho = Rhosuco = \frac{E_c}{J_c} * \left|\frac{B_{z\_emqap}}{J_c}\right|^{-1+n} + 10^{-18} ,$$
$$Rho = Rhoair = 10^{-5} . \qquad (4)$$

The external (transport) current is set as a variable I_1. The actual current density is integrated over the HTS domain. This scalar value Iext is then balanced with the preset current in this point: I_1-I_{ext}=0. The location of this point is irrelevant. It can be completely outside the active geometry. In the COMSOL GUI it appears as shown in Fig. 2.

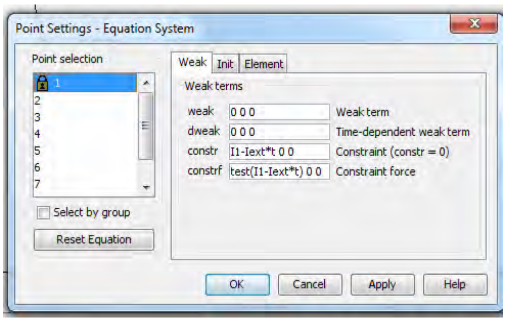

Figure 2. Point settings in the Equation System.

When needed, external (e.g. perpendicular to the tape broad face) magnetic field can be applied by proper selection of the boundary conditions (e.g. by setting in Physics, Boundary settings, Boundary condition to: "Magnetic potential" and providing to the Quantity A_0 a certain value or expression).

Despite the open character of COMSOL, not all variables can be directly accessed at present. A few background calculations are performed for vector edge elements. Some specific boundary conditions however typically rely on these variables. In such cases workarounds may be needed as explained below.

4. Application examples

Strip geometry. In all cases below a filament or a tape with rectangular cross-section (a strip) is used as shown in Fig. 3. The strip is infinite in the z-direction. The external magnetic field is applied in the y-direction, the currents flow in the z-direction. In practice, most of YBCO tapes are 2 to 10 mm wide, their thickness *t* (in y-direction) varies between 50 and 100 μm.

The superconducting YBCO layer is typically 1 μm thick. The superconductor aspect ratio is large in a tape and it creates a problem for meshing. The filaments are typically less than 1 mm wide. For example, a striated 12 mm wide tape used below consists of ten 0.84 mm wide filaments stacked together in x-direction and separated from each other with 0.4 mm wide gaps. Where suitable, the symmetries as described in [1] are used to speed up the computation. Other specifications of the model strips are listed in Table 3.

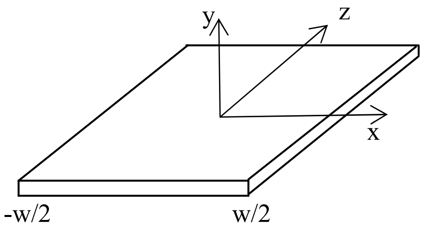

Figure 3. Geometry of the YBCO strip.

Table 3: Specifications of the strips used in this paper

Sample nr.	1	2a	2b	3
W, [mm]	12	0.84	0.84	4
t, [μm]	50	50	10	50
$J_c \cdot 10^{-8}$[A/m^2]	3.8	3.6	18	16
N	26	26	26	26

Meshing of very thin and wide objects is a problem in COMSOL. The aspect ratio of the superconducting layer can be as high as 10^4. In this study we assume a superconducting layer being 50 μm thick (except for the strip nr. 2b, Table 3) and use a triangular mesh with a refinement, see example in Fig. 4.

The AC losses (in W/m) in a period *T* are computed over the HTS tape domain as:

$$P = \frac{1}{T}\int_o^T dt \int_S J_z^2 \rho(J_z) dS \ . \qquad (5)$$

4.1 HTS strip in external perpendicular magnetic field

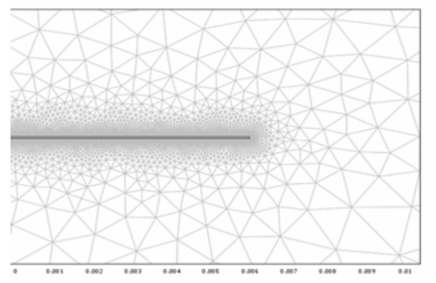

Figure 4. Example of a mesh for 12 mm-wide YBCO tape (only part of the actual tape cross-section centered around zero is shown in the figure).

Specifications of the two modeled strips nr. 1 and 2 are listed in Table 3, example of the mesh is shown in Fig. 4. Magnetic field is applied by the proper selection of boundary conditions (by selecting Physics, Boundary settings, setting Boundary condition to: “Magnetic potential” and providing to Quantity A_0 a certain value or expression, in our case: $A_{y0}\cdot\sin(2\pi\cdot f\cdot t)$), with *f* being a frequency.

In Fig. 5 calculated AC losses for a sample strip nr. 1 and for ten separate strips nr. 2a, Table 3 (the computed points are shown by the circles and by the triangles respectively) are compared to the theory of Brandt [3] (shown by the dashed lines) and a good agreement is found, the errors are within a few percent over the entire range.

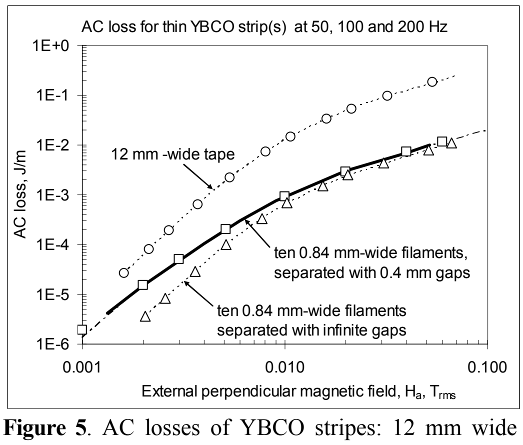

Figure 5. AC losses of YBCO stripes: 12 mm wide tape; ten separate filaments; a finite array of ten filaments separated by gaps of 0.4 mm; the dashed lines: analytical results of [3] and [4]; the solid line: experiment [5]; the circles, the triangles and the rectangles are the points computed with COMSOL for the same conditions.

Validation of the model is performed by comparing computed magnetic field and current density distributions over the strip width with the theory of Brandt [3]. The comparison shows a fair agreement of the computed and analytical profiles.

4.2 HTS strip with a transport current

Specifications of the model strips nr. 2a and 2b are listed in Table 3.

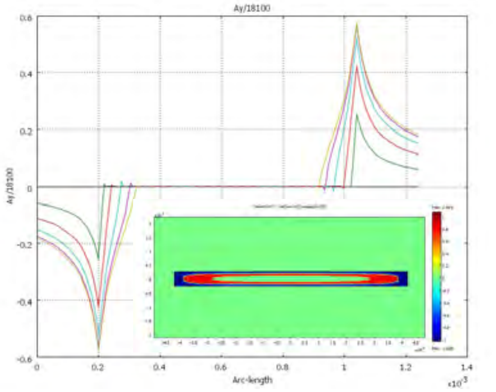

Figure 6. Computed magnetic field distributions over the width of the strip nr. 2a, scaled current distribution J_z/J_c is also shown.

Validation of the model is performed by comparing computed magnetic field and current density distributions over the strip width with those given by the theory [3], again a fair agreement is found. For the 50 Hz sinusoidal transport current examples of the computed magnetic field profiles (plotted at 1, 2, 3, 4 and 5 ms) and the current density distribution (at 30 ms) are shown in Fig. 6 for the strip 2a.

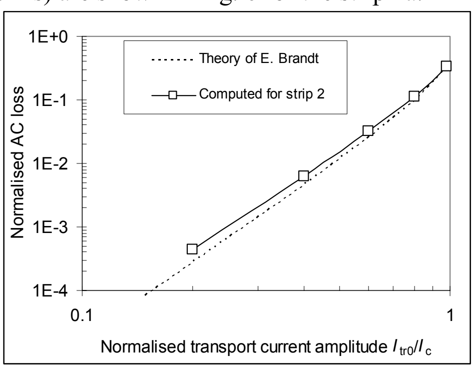

Figure 7. Computed AC losses as a function of the transport current for a thin strip 2b (boxes), compared to the theory [3] (dashed line). The losses are normalized by the factor $fI_c^2\mu_0/\pi$.

Furthermore, in Fig. 7 calculated AC losses of the strip with 50 Hz sinusoidal transport current are compared to the theory of Brandt for a thin strip [3] and the agreement is reasonable. Observed deviation between theory and computation at lower current amplitudes will be addressed elsewhere.

4.3 Array of HTS strips (x-stack)

For infinite arrays the symmetries [2] where applicable are used to speed up the computation process, here below we present an example of COMSOL computation for a finite *x*-stack of YBCO strips. In this case an array of ten filaments is made by striating a 12 mm wide tape into the array of 0.84 mm-wide filaments separated by gaps of 0.4 mm [5]. Specifications of the comprising strips nr.2a are listed in Table 3. Using the symmetries, one quarter of the actual cross-section shown in Fig. 8 can be modeled. Example of the mesh is also shown in the figure.

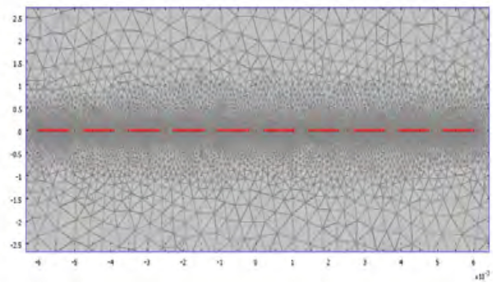

Figure 8. Example of the mesh used for the 12 mm-wide striated YBCO tape.

AC losses are computed for the case when the array is in external perpendicular magnetic field oscillating at 50 Hz. The results are compared to the theory [4] and to the experiment [5] as shown in Fig. 6. Here the boxes represent the computed points, the solid line represents the experiment [5] performed at 100 Hz and 200 Hz and the dashed line represents the theory [4]. Our conclusion is that computation results are in excellent agreement with both the experiment and the theory over the entire range of magnetic field amplitudes. Thereby, this model (of striated tape) is also validated. In addition, Fig. 9 shows an example of the computed current distribution J_z/J_c inside one of the middle filaments of the striated tape.

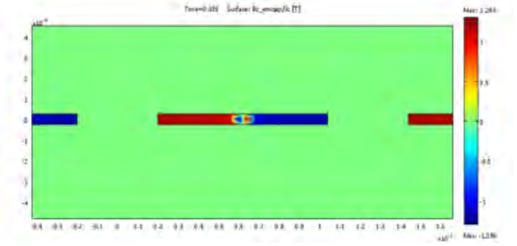

Figure 10. Example of the computed current distribution Jz/Jc inside one of the filaments of the striated tape (at 50 Hz, 30 ms, and 60 mTrms).

4.4 AC HTS cable (single phase)

Finally, we use COMSOL to model a HTS cable made of YBCO tapes. Example of a simplified model for a single layer, single phase HTS cable made in this case with eight coated tapes assembled into a polygon cable conductor [6] is shown in Fig. 10. Naturally, depending on the diameter of the cable former, on the tape width, number of layers, etc., a different number of tapes can be in the polygon cable conductor, see for instance [7].

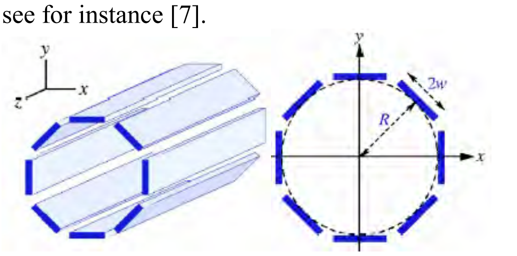

Figure 10. Model geometry for a single layer cable made of coated conductor tapes, after [6].

Because YBCO tapes are thin and in a cable conductor number of the tapes can be substantial, the computation process of HTS cable can be time consuming.

In order to speed up the computation process, one can use the symmetry considerations and model a sector that includes just a few tapes or even a (part of a) singe tape as further illustrated in Fig. 11for the case of single layer model cable. In this case a symmetry axis is the line connecting the center of the cable former with the middle point of the gap between two adjacent tapes, case A (and/or with the middle of the tape, case B). In the case A, Neumann boundary condition is set along the symmetry lines, and in the case B it is the condition $H_y = 0$.

Using this symmetry approach, one can compute AC losses in a single layer polygon

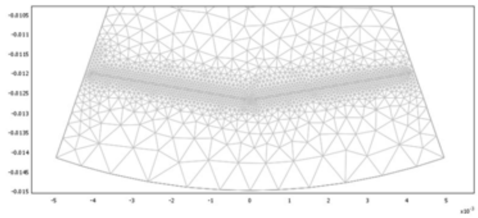

Figure 11. Example of the symmetry use in COMSOL geometry for a single layer polygon cable conductor.

conductor comprised for instance of 19 tapes separated by 0.2 mm gaps (for the case shown in the figure) by modeling just one or two tapes (nr. 3, Table 3) and then multiplying the AC losses by the number of tapes. Example of the AC losses computed this way is shown in Fig. 12. Here the critical current of each tape is 320 A, the critical current of the cable is about 6 kA.

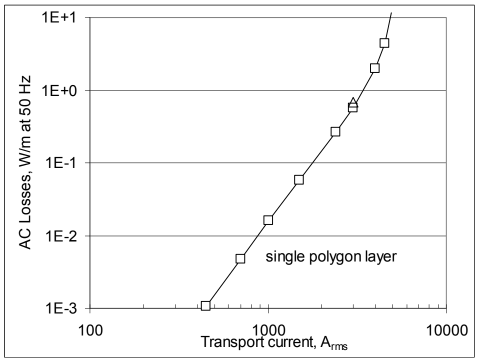

Figure 12. Computed AC loss of a model single-layer and single-phase YBCO cable made of nineteen 4-mm wide YBCO tapes separated by 0.2 mm gaps and placed around a 25 mm diameter former.

Using the approach, it is possible to model more complex structures (e.g., a cable with more tape layers, a three-phase cable, a triax cable, etc.), all this together with the relevant experimental data and detailed validations will be reported elsewhere.

7. Conclusions

We developed and validated a set of numerical models for coated conductor HTS tapes, arrays and cables using COMSOL Multiphysics. Since existing COMSOL example from AC/DC module is used, the changes in COMSOL relevant to modeling of HTS tapes are kept to the minimum. A user aiming to model HTS tapes and devices with COMSOL, can access the modeling process more directly, without a need to write PDE's first.

8. References

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9. Acknowledgements

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