

# Modeling of coupled hydro-mechanical processes occurring during CO<sub>2</sub> injection – example from In-Salah

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# Agenda

- Introduction to two-phase flow modeling
  - Two-phase flow equations
    - Various formulations
    - Comparison, weaknesses, strengths, challenges
- Carbon Capture and Storage – CCS
  - Modelling of coupled hydro-mechanical processes occurring during CO<sub>2</sub> injection - example from In Salah

## Two-phase flow equations

$$\frac{\partial}{\partial t}(\phi \rho_{\alpha} S_{\alpha}) + \nabla \cdot (\rho_{\alpha} \mathbf{u}_{\alpha}) = \rho_{\alpha} q_{\alpha} \quad \text{Mass balance}$$

$$\mathbf{u}_{\alpha} = -\frac{k_{r\alpha}}{\mu_{\alpha}} \mathbf{K} (\nabla p_{\alpha} - \rho_{\alpha} \mathbf{g}) \quad \text{Darcy velocity}$$

$$\sum S_{\alpha} = 1, \quad p_c = p_n - p_w, \quad S_{e\alpha} = p_c(S_{\alpha}) \quad \text{Auxiliary equations}$$

Two immiscible fluids in saturated porous media

## Two-phase flow equations

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But... there's also the global/total pressure!  
Relating the partial pressures, here showing  
Three definitions...

## Various definitions of global/total pressure $p_s$ :

1. Flooding:  $p_s = p_w + p_n$

2. Weighted:  $p_s = S_w p_w + S_n p_n$

3. Fractional:  $p_s = p_n - \int_S (f_w \frac{dp_c}{dS})(\xi) d\xi = p_w + \int_S (f_n \frac{dp_c}{dS})(\xi) d\xi$

leading to relations between the derivatives;

$$\nabla p_s = \nabla p_w + f_n \cdot \nabla p_c = \nabla p_n - f_w \cdot \nabla p_c$$

# Some examples of two-phase flow eq.

- By manipulating the mass balances, one can obtain many different formulations. Two main groups:
- **Pressure based:**
  1. Partial pressure:  $p_w - p_n$
  2. Flooding:  $p_s - p_c$
- **Saturation based:**
  3. Pressure saturation:  $p_w - S_n$
  4. Pressure saturation:  $p_n - S_w$
  5. Fractional flow:  $p_s - S_w$
  6. Fractional flow:  $p_s - S_n$

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Global/total pressure:

**6**  $\Rightarrow$  **12**  $\Rightarrow$  **18**

formulations

Assumptions and simplifications:

- Homogeneous and isotropic media
- Compressibility
- Gravity

## Deriving the equations...

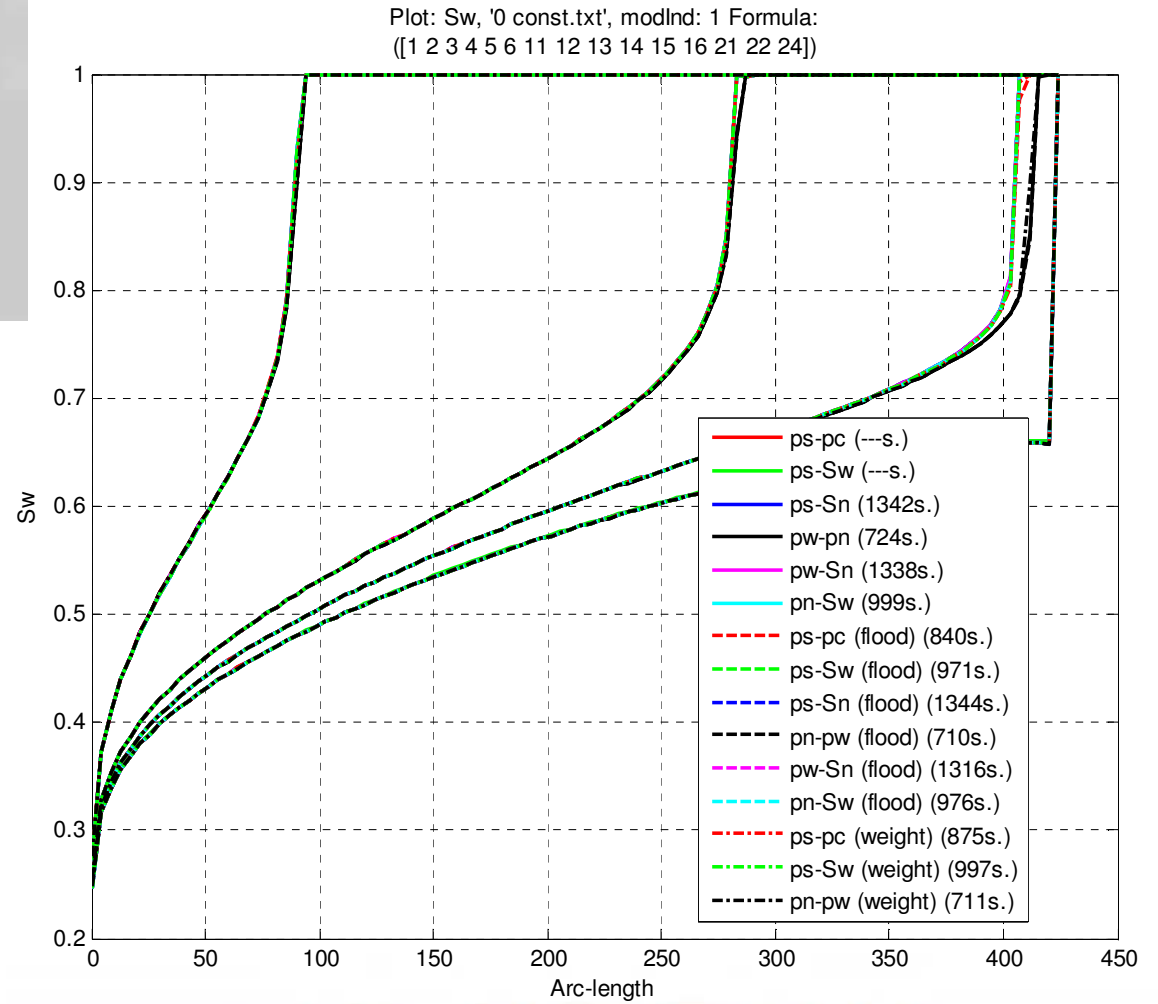
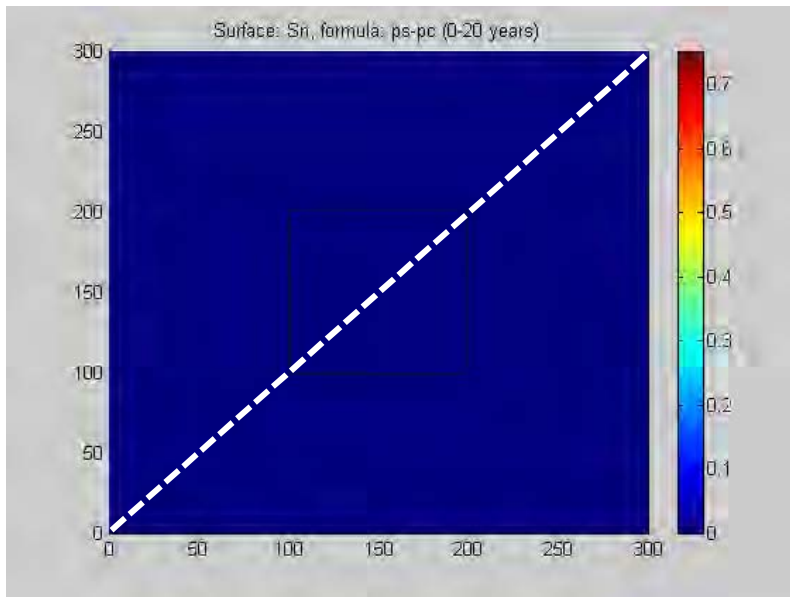
- Using any definition of  $p_s$ :
- $p_w$ -equation:

$$-\phi \frac{\partial S_w}{\partial p_c} \frac{\partial p_w}{\partial t} - \nabla \cdot \frac{k_{rw}}{\mu_w} \mathbf{K} \nabla p_w = -\phi \frac{\partial S_w}{\partial p_c} \frac{\partial p_n}{\partial t}$$

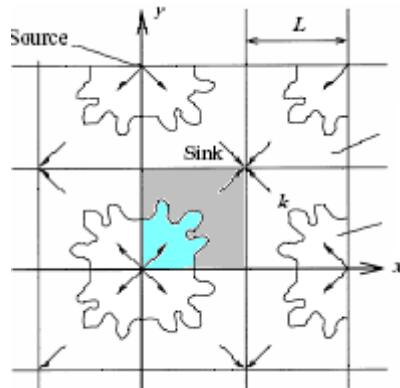
- $p_n$ -equation:

$$-\phi \frac{\partial S_w}{\partial p_c} \frac{\partial p_n}{\partial t} - \nabla \cdot \frac{k_{rn}}{\mu_n} \mathbf{K} \nabla p_n = -\phi \frac{\partial S_w}{\partial p_c} \frac{\partial p_w}{\partial t}$$





Five spot model:



Simple and straight forward!

Assumptions and simplifications:

- Homogeneous and isotropic media
- Compressibility
- Gravity

## Deriving the equations...

- Using the fractional definition:

$$\nabla p_s = \nabla p_w + f_n \cdot \nabla p_c = \nabla p_n - f_w \cdot \nabla p_c$$

- $\rho_s$ -equation:

$$\begin{aligned} & (\phi(S_n \rho_n c_{f,n} + S_w \rho_w c_{f,w}) + (\rho_n - \rho_n S_w + \rho_w S_w) \phi^0 c_R) \frac{\partial p_s}{\partial t} + \nabla \cdot ((K_w f_n - K_n f_w) \frac{\partial p_c}{\partial S_w} \nabla S_w - \\ & (K_n + K_w) \nabla p_s + (K_n \rho_n + K_w \rho_w) \mathbf{g}) = (\phi(\rho_w - \rho_n) + \phi(S_n \rho_n c_{f,n} f_w - S_w \rho_w c_{f,w} f_n) \frac{\partial p_c}{\partial S_w}) \frac{\partial S_w}{\partial t} \end{aligned}$$

- $S_w$ -equation:

$$\phi \rho_w (1 - S_w c_{f,w} f_n) \frac{\partial p_c}{\partial S_w} \frac{\partial S_w}{\partial t} - \nabla \cdot (K_w (\nabla p_s - f_n \frac{\partial p_c}{\partial S_w} \nabla S_w - \rho_w \mathbf{g})) = -S_w \rho_w (\phi c_{f,w} + \phi^0 c_R) \frac{\partial p_s}{\partial t}$$

Assumptions and simplifications:

- Homogeneous and isotropic media
- Compressibility
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## Deriving the equations...

- Using the fractional definition:

$$\nabla p_s = \nabla p_w + f_n \cdot \nabla p_c = \nabla p_n - f_w \cdot \nabla p_c$$

- $\rho_s$ -equation:

$$\begin{aligned} & (\phi(S_n \rho_n c_{f,n} + S_w \rho_w c_{f,w}) + (\rho_n - \rho_w S_w + \rho_w S_w \phi^0 c_R) \frac{\partial p_s}{\partial t} + \nabla \cdot ((K_w f_n - K_n f_w) \frac{\partial p_c}{\partial S_w} \nabla S_w - \\ & (K_n + K_w) \nabla p_s + (K_n \rho_n + K_w \rho_w) \mathbf{g}) = (\phi(\rho_w - \rho_n) + \phi(S_n \rho_n c_{f,n} f_w - S_w \rho_w c_{f,w} f_n) \frac{\partial p_c}{\partial S_w}) \frac{\partial S_w}{\partial t} \end{aligned}$$

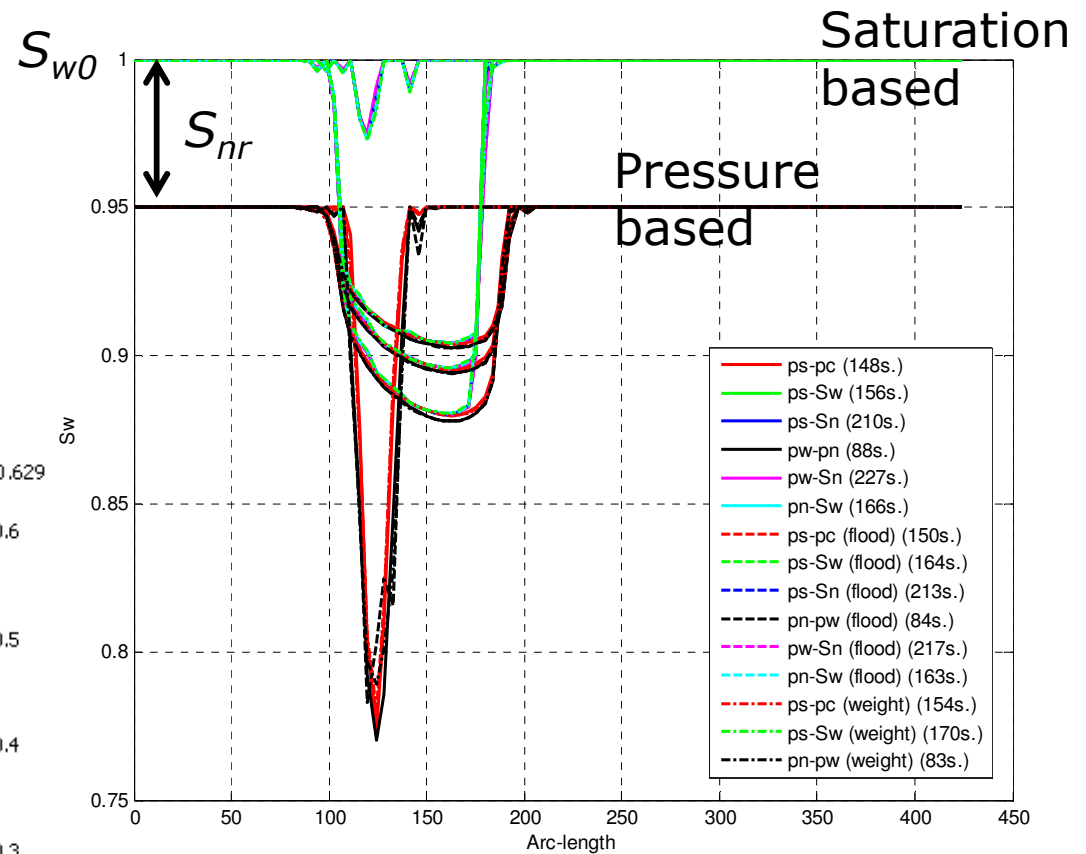
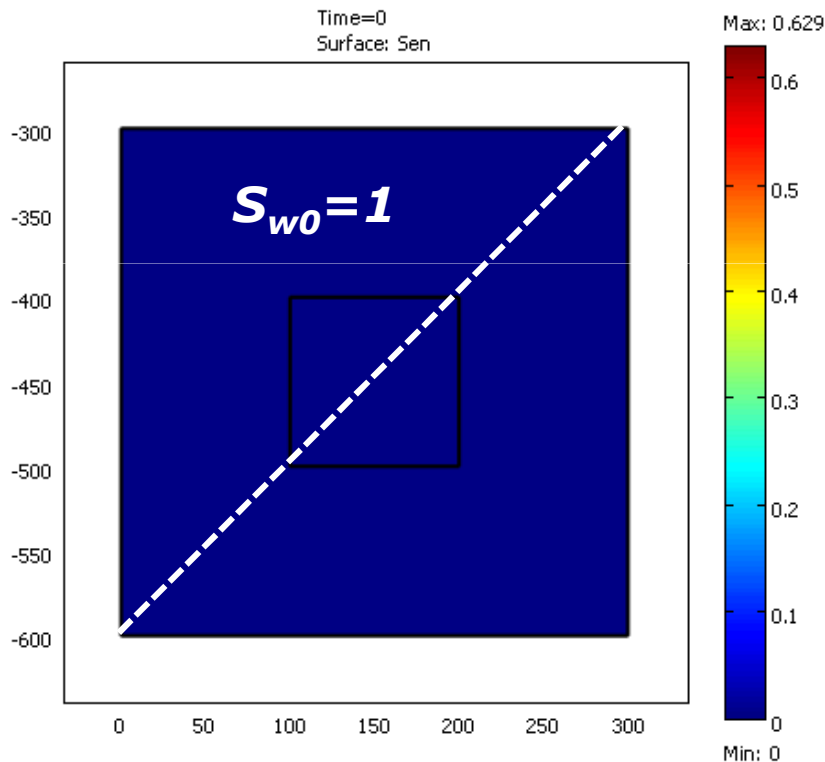
- $S_w$ -equation:

$$\phi \rho_w (1 - S_w \phi_{f,w} f_n \frac{\partial p_c}{\partial S_w}) \frac{\partial S_w}{\partial t} - \nabla \cdot (K_w (\nabla p_s - f_n \frac{\partial p_c}{\partial S_w} \nabla S_w - \rho_w \mathbf{g})) = -S_w \rho_w (\phi_{f,w} + \phi^0 c_R) \frac{\partial p_s}{\partial t}$$

Beware of artificial "source-terms" in pressure based forms!!<sub>11</sub>

Saturation based:  
Gives  $S_w/S_n$  directly:

$$S_{wr} \leq S_w \leq S_{w0}$$



Pressure based gives  $S_w/S_n$  through capillary pressure functions:

$$S_{ew} = S_{ew}(p_c)$$

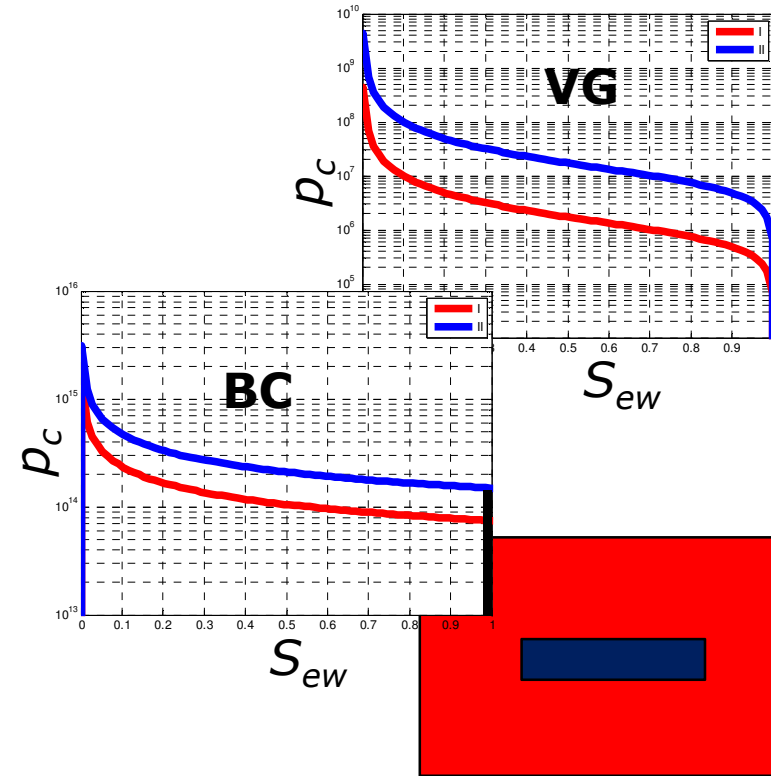
$$S_w = S_{ew}(1 - S_{wr} - S_{nr}) + S_{wr}$$

$$S_{wr} \leq S_w \leq 1 - S_{nr} (\leq S_{w0})$$

# Heterogeneous reservoir...

- A whole different story; when two neighboring domains have different capillary functions:

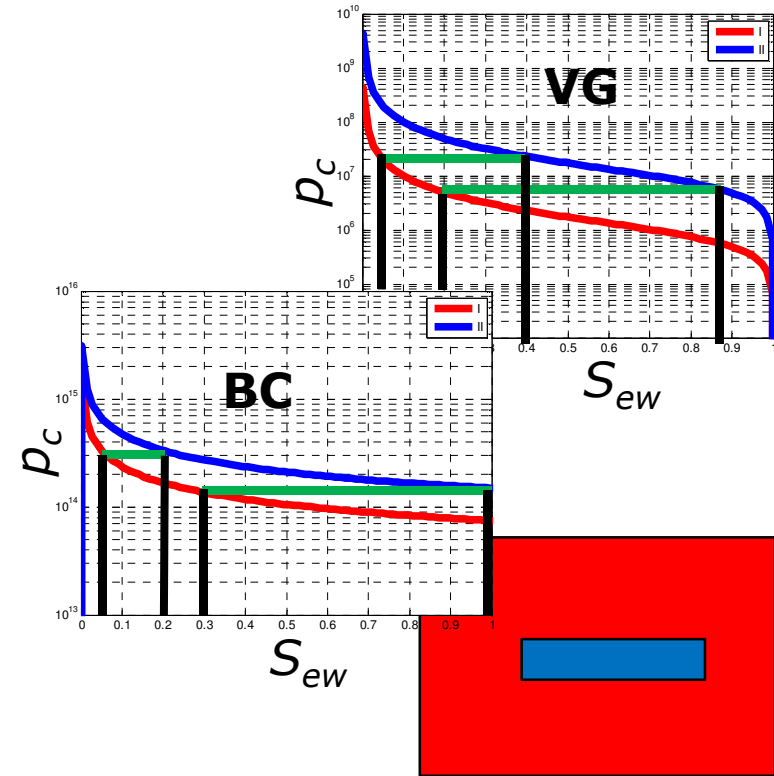
1. Continuity in flux: the flux of both phases have to be continuous across the interface
2. Continuity in capillary pressure, and the phase pressure that is mobile on both sides of the interface
3. Discontinuous phase saturations



# Heterogeneous reservoir...

- A whole different story; when two neighboring domains have different capillary functions:

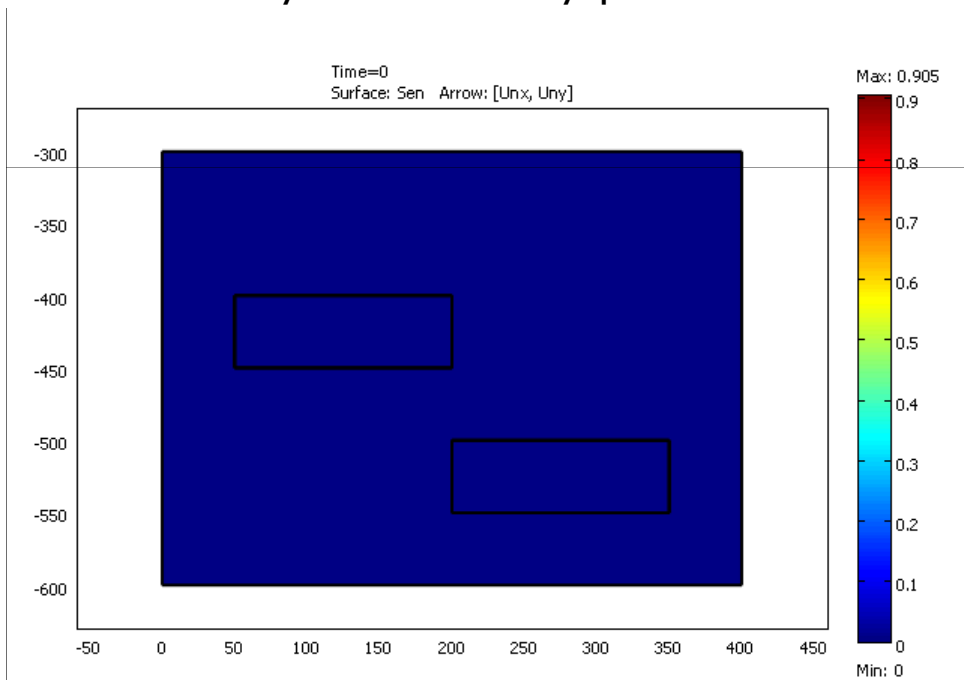
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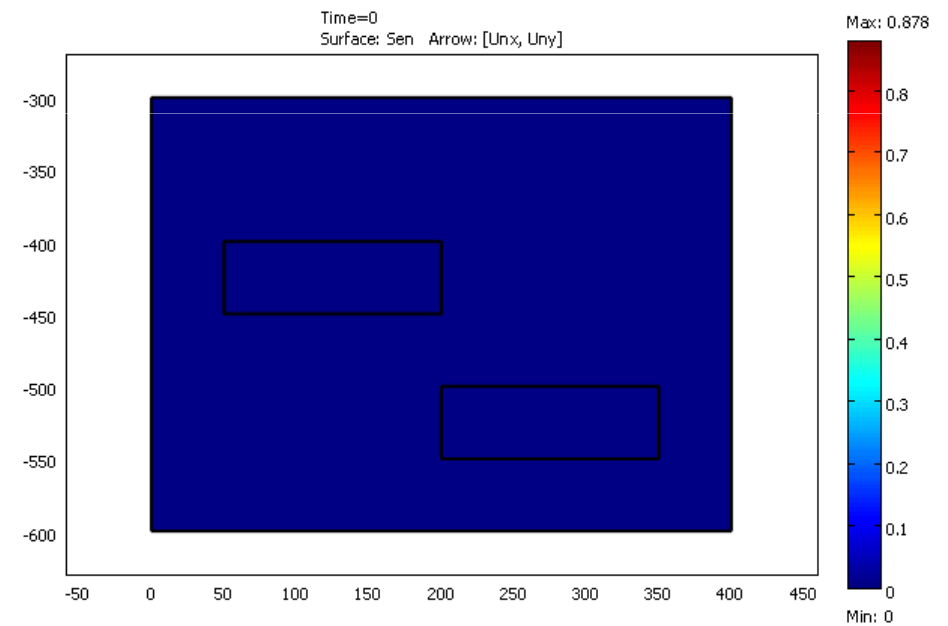
# Discontinuity:

## Entry pressure VS permeability

Discontinuity due to entry pressure



Discontinuity due to Permeability



# Summary

- Easy to derive a set of equations for two-phase flow
- Heterogeneous media can be “easily” handled:
  - Constraining the saturations on opposite sides of an interface to a capillary function ratio
- $p_n - S_w$  and  $p_w - S_n$  are robust candidates for two-phase flow modeling
  - Handles discontinuities, also faults and fractures, and residual saturations, but lack speed
- Pressure based are the fastest
  - But, least robust as it's lacking some important features; support for residual saturations and heterogeneities



# Carbon Capture and Storage - CCS

Example from In-Salah, Algeria



# Introduction

- At In Salah, Algeria, excess CO<sub>2</sub> from the produced oil and gas is re-injected (app. 1 mill. tons/year) into the ground as part of a CO<sub>2</sub> storage demonstration project
- A significant heave at the injection sites was observed (InSAR): 5-8 mm/yr (as much as 15 mm after 3 years)
  - Several kilometers footprint. Modeling has verified the observations
- Still, we wanted to apply our model to a “realistic” case:
  - For “verification” and see if there is any lessons to be learned

# Biot linear poroelasticity equation - short

- Linear Biot poroelasticity:
  - Elastic response of fluid saturated porous media; linear elastic solids undergoing quasistatic small deformations:

$$\nabla \cdot [\boldsymbol{\sigma}] = -\mathbf{F}(\rho_{f0})$$

$$\boldsymbol{\sigma} = \mathbf{D}_d \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_0$$

# Biot linear poroelasticity equation - short

- Linear Biot poroelasticity:
  - Elastic response of fluid saturated porous solid; linear elastic solids undergoing quasistatic small deformations

$$\begin{array}{ccc}
 \nabla \cdot [\boldsymbol{\sigma}] = -\mathbf{F}(\rho_f) & \xleftrightarrow{\text{Coupling}} & Q = -\alpha_{biot} \frac{\partial \varepsilon_v}{\partial t} \\
 \boldsymbol{\sigma} = \mathbf{D}_d \boldsymbol{\varepsilon} + \boldsymbol{\sigma}_0 - \alpha_{biot} p \mathbf{l} & & \phi = (1 - \varepsilon_v) \phi^0
 \end{array}$$

$$\phi \rho_w (1 - S_w c_{f,w} f_n \frac{\partial p_c}{\partial S_w}) \frac{\partial S_w}{\partial t} - \nabla \cdot (K_w (\nabla p_s - f_n \frac{\partial p_c}{\partial S_w} \nabla S_w - \rho_w \mathbf{g})) = q_w \rho_w - S_w \rho_w (\phi c_{f,w} + \phi^0 c_R) \frac{\partial p_s}{\partial t}$$

# Studies to learn some lessons

## 1. Base case

- Simplified, best guess model

## 2. Fracture case

- High-permeable lower-caprock

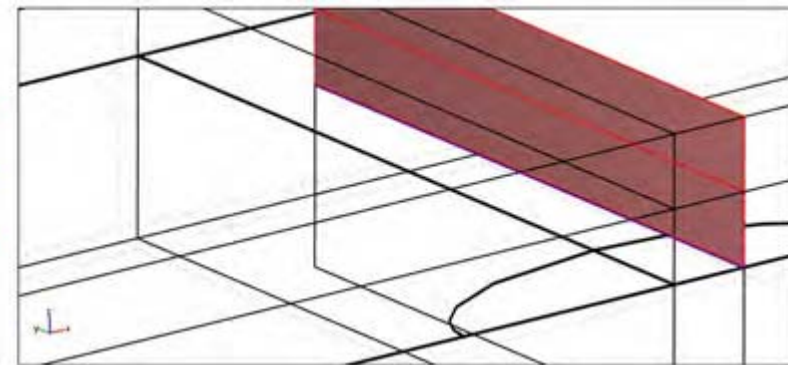
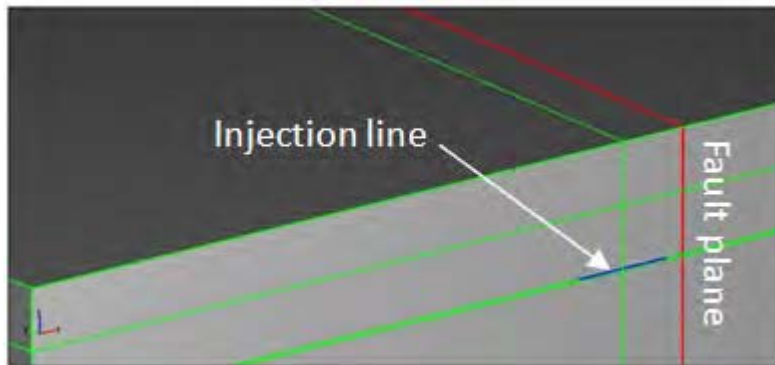
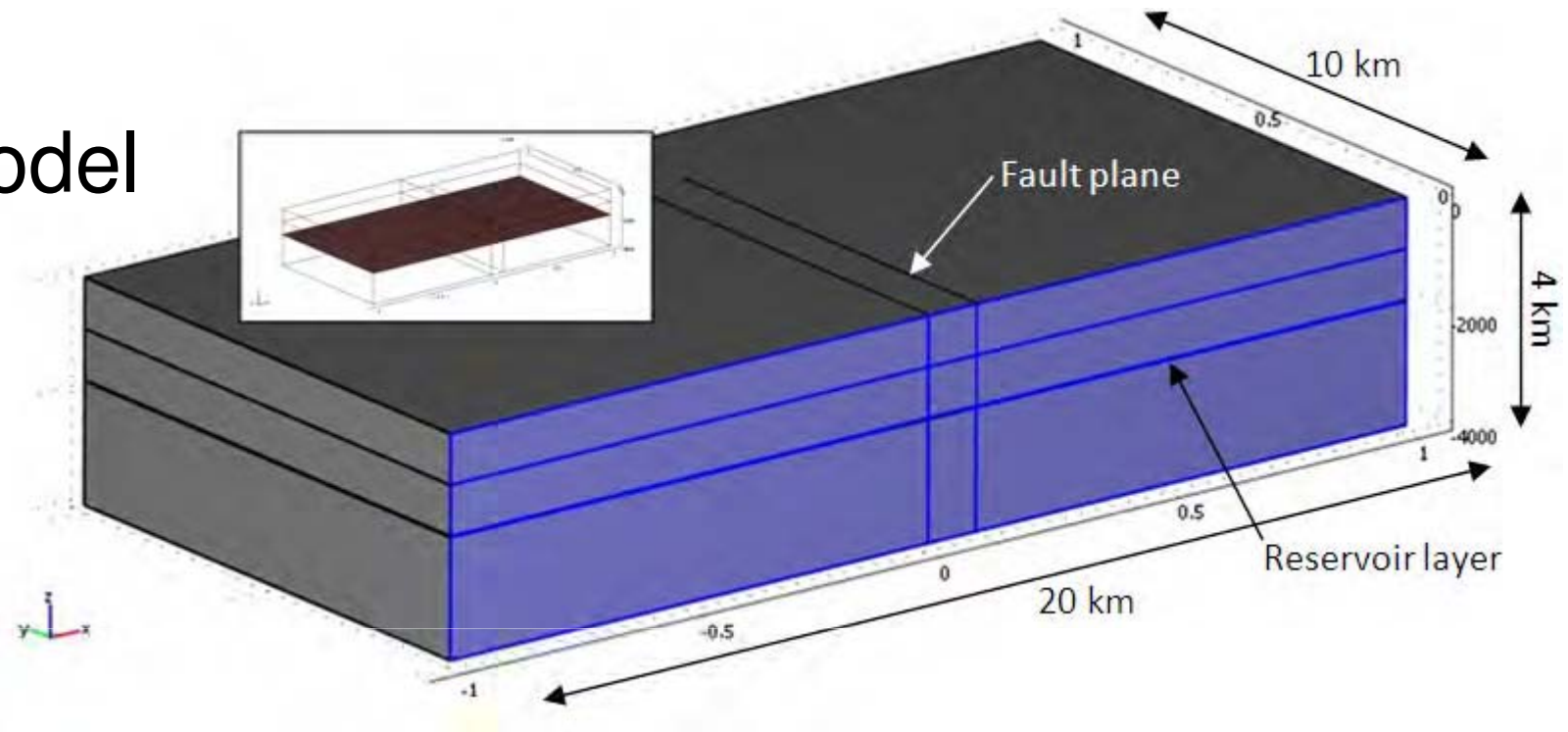
## 3. Fault case

- Best guess model with a vertical fracture/fault plane intersecting the caprock

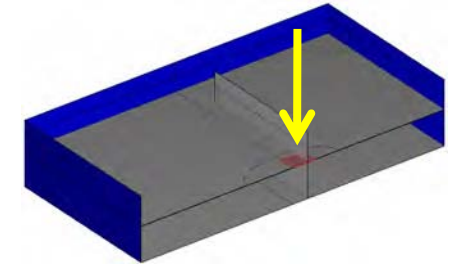
Layer	Depth, [m] (thickness, [m])	Hydraulic properties	Elastic properties
Cretaceous sandstone and mudstone overburden	0-900 (900)	$K = 10^{-19} \text{ m}^2$ $\phi = 0.17$ $\rho_{fl} = 1 \text{ MPa}$	$E = 7 \text{ GPa}$ $\nu = 0.15$
Carboniferous mudstone	900-1800 (900)	$K = 10^{-19} \text{ m}^2$ $\phi = 0.17$ $\rho_{fl} = 1 \text{ MPa}$	$E = 7 \text{ GPa}$ $\nu = 0.15$
(High permeable, lower caprock)	1640-1800 (160)	$(K = 200 \text{ mD}, \phi = 0.17, \rho_{fl} = 1 \text{ MPa})$	$(E = 6 \text{ GPa}, \nu = 0.2)$
C10.2 Sandstone	1800-1820 (20)	$K_{eff} = 200 \text{ mD}$ $\phi = 0.15-0.2 (\approx 0.17)$ $\rho_{fl} = 1 \text{ MPa}$	$E = 6 \text{ GPa}$ $\nu = 0.2$
D70 mudstone underburden	1820-4000 (2180)	$K = 10^{-19} \text{ m}^2$ $\phi = 0.17$ $\rho_{fl} = 1 \text{ MPa}$	$E < 5 \text{ GPa}$ $\nu = 0.15$

Diagram features: A vertical dashed red line labeled "Fault line" is located at approximately 1640 m depth. A horizontal dashed blue line labeled "Injection line" is located at approximately 1800 m depth. A yellow inverted triangle is in the top right corner of the table area.

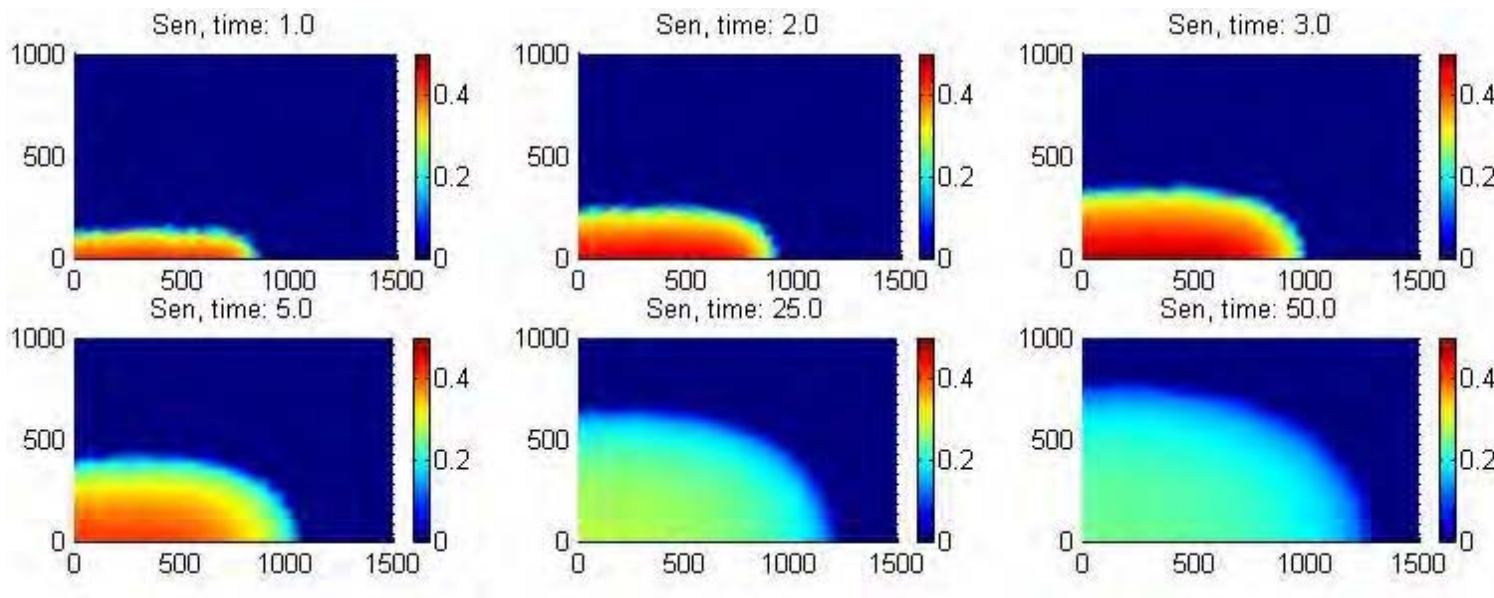
# Model



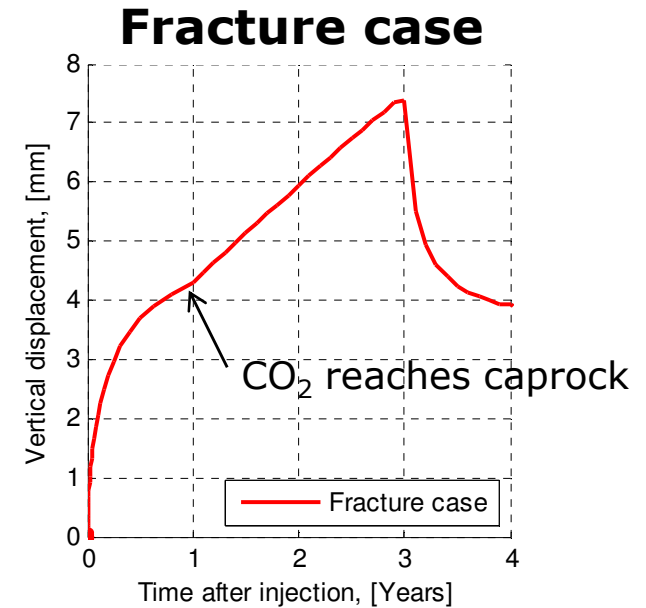
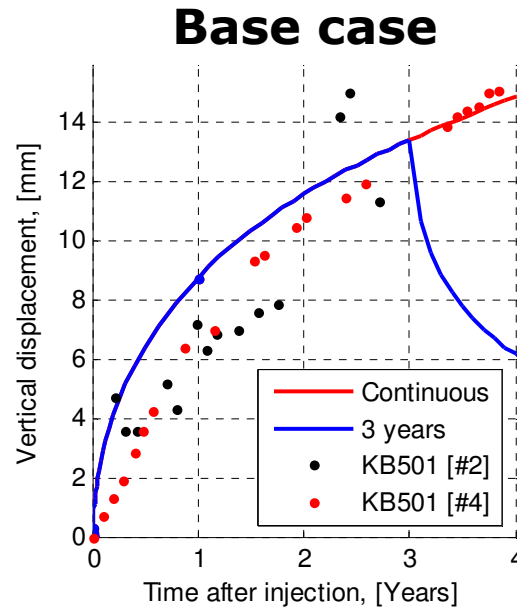
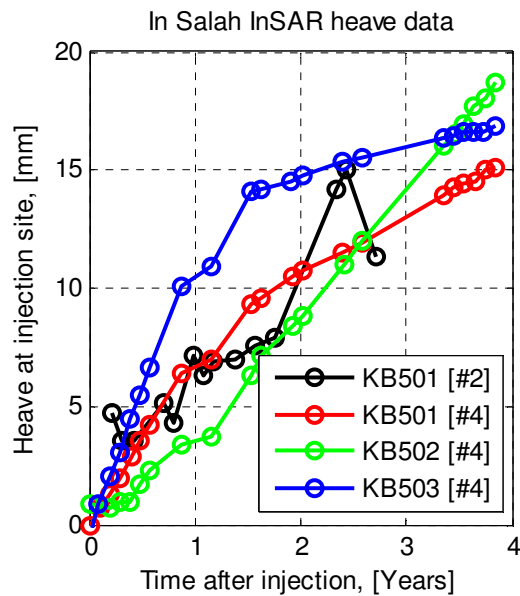
# Results (base case)



- The base case model is solved for injection of CO<sub>2</sub> over 3 years and 50 years



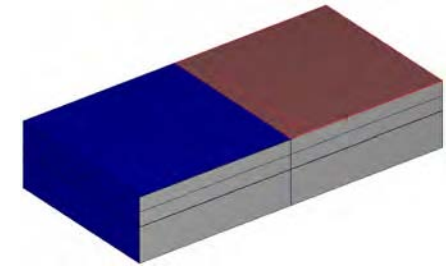
Snapshots of the CO<sub>2</sub> plume at various times (seen from above, along the top surface of the reservoir; -1800 m).



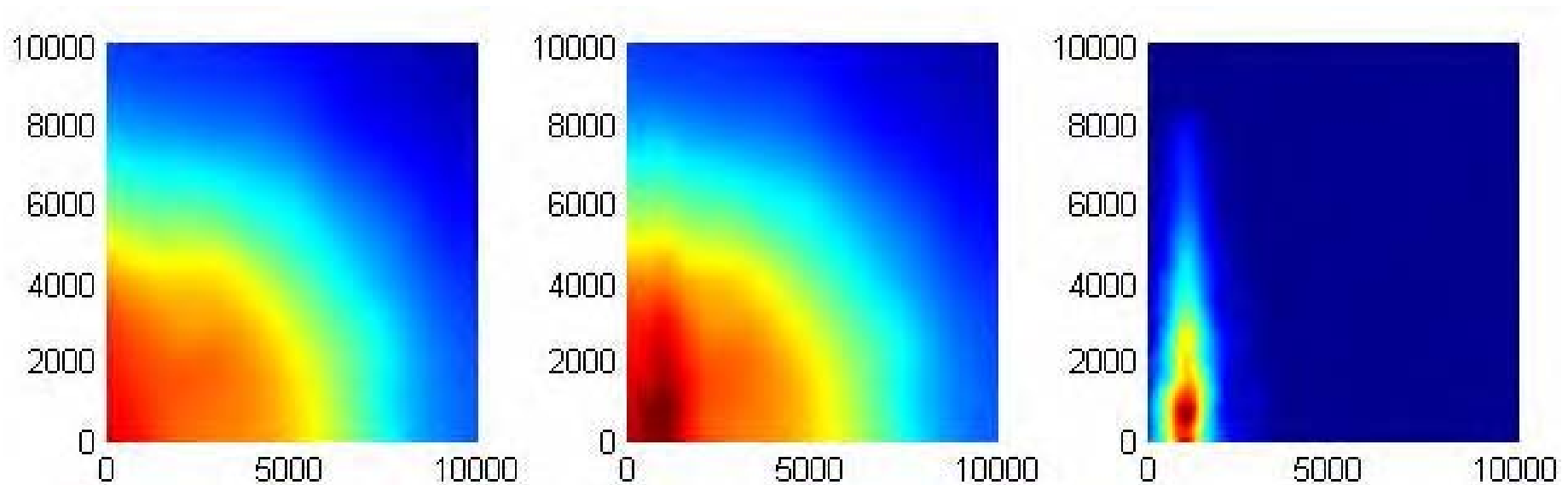
- Comparing modeled vs. measured:
  - Good agreement (except fractured case)
- Comparing cases:
  - Heave is not comparable, however, the shape is interesting and may say something about the hydraulic properties



# Results, fault case

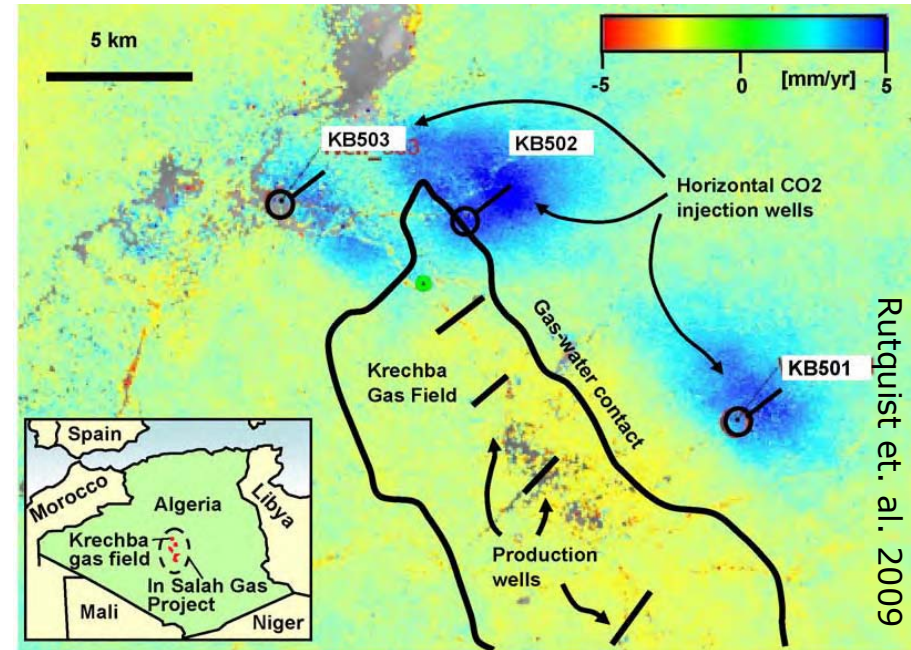


- Snap shots vertical displacement after 3 years. Color scale is 0-15 mm, in difference plot: 0-3 mm

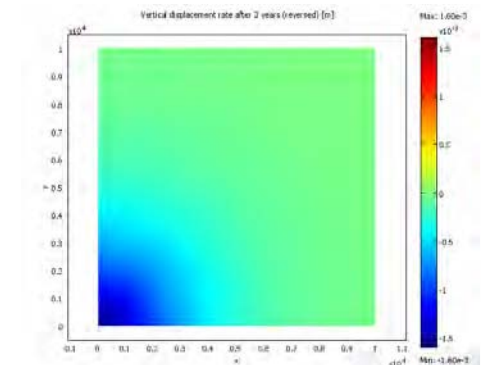
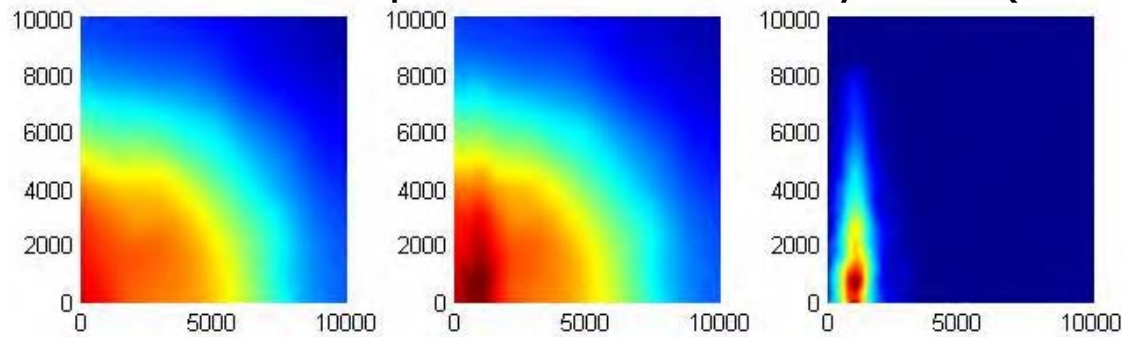


The heave is **not** due to leakage through the fault.

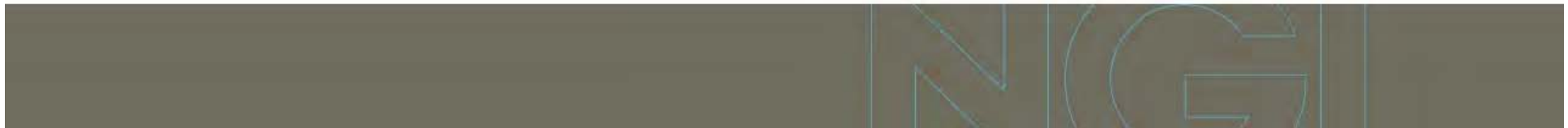
- Fault case:  
A large fault can give a distinct heave pattern at the surface, even out of reach from the plume



Total vertical displacement after 3 years. (0-15mm and 0-3 mm)



Rate of vertical displacement after 3 years (-1.6-1.6 mm/yr)



# Summary

- Even a simple model is able to capture the main effects of a real case
- The shape of the heave curve at the top surface can say something about the geology and hydraulic properties
  - Here a fractured zone above the injection layer
- Fault/fracture planes give visible footprints on the surface and whether it behaves as a seal or a conduit for flow