Scattering of Electromagnetic Waves by Particles

Particles can be characterized by the unique scattering patterns produced by their interaction with electromagnetic waves. Optical scattering measurements cover a broad range of applications such as meteorology, particle sizing, biomedical, and metamaterials.

BY SERGEI YUSHANOV, JEFFREY S. CROMPTON, AND KYLE C. KOPPENHOEFER, ALTASIM TECHNOLOGIES

As electromagnetic waves propagate through matter they interact with particles or inhomogeneities that perturb the local electron distribution. This variation produces periodic charge separation within the particle, causing oscillation of the induced local dipole moment. This periodic acceleration acts as a source of electromagnetic radiation, thus causing scattering.

PARTICLE SIZE MATTERS

Scattering of electromagnetic waves by particles can be illustrated by two theoretical frameworks: Rayleigh scattering that is applicable to small, dielectric, non-absorbing spherical particles, and Mie scattering that provides a general solution to scattering that is independent of particle size. Mie scattering theory converges to the limit of geometric optics at large particle sizes. Consequently, Mie scattering theory can be used to describe most scattering by spherical particles, including Rayleigh scattering, but due to the complexity of implementation, Rayleigh scattering theory is often preferred.

The Rayleigh scattering model breaks down when the particle size becomes larger than approximately 10 percent of the wavelength of the incident radiation, at which point Mie theory must be applied. The Mie solution is obtained by analytically solving Maxwell's equations for the scattering of electromagnetic radiation by spherical particles; it is modeled in terms of infinite series rather than a simple mathematical expression.

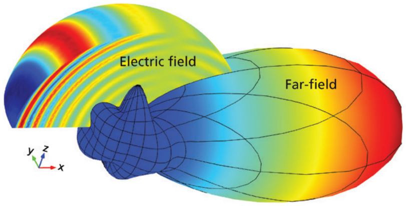

FIGURE 1. Electric field due to Mie scattering of the incident wave in the x-direction showing enhanced scattering in the forward direction.

Mie scattering differs from Rayleigh scattering in several respects: it is mostly independent of wavelength and is larger in the forward direction than in the reverse direction (see Figure 1). The greater the particle size, the more light is scattered forward. In addition to the many atmospheric effects of light scattering, applications of Mie scattering include environmental models such as dust particles in the atmosphere and oil droplets in water, as well as medical technology used to measure cell nuclei in biological systems or the collagen fibers in body tissue.

MIE SCATTERING

Implementation of analytical solutions for Mie scattering by a particle or object is complex and requires solving Maxwell's equations to represent the incident, scattered, and internal fields. These take the form of infinite series expansion of vector spherical harmonics, allowing the cross sections, efficiency factors, and distributions of intensity to be predicted. Further, the influence of particle geometry, the angle of incidence of the wave, and the particle material properties can be investigated.

In electromagnetic wave scattering problems, the total wave decomposes into the incident and scattered wave components. Important physical quantities can be obtained from the scattered fields. One of these is the cross section, which can be defined as the net rate at which electromagnetic energy crosses the surface of an imaginary sphere centered at the particle, divided by the incident irradiation (P_{inc}). To quantify the rate of the electromagnetic energy absorbed (W_{abs}) and scattered (W_{sca}) by the particle, the absorption (σ_{abs}), scattering (σ_{sca}), and extinction (σ_{ext}) cross sections are defined as:

$$\sigma_{abs} = \frac{W_{abs}}{P_{inc}}, \quad \sigma_{sca} = \frac{W_{sca}}{P_{inc}}, \quad \sigma_{ext} = \sigma_{abs} + \sigma_{sca}$$

The total absorbed energy is derived by integrating the energy loss over the volume of the particle. The scattered energy is derived by integrating the Poynting vector over an imaginary sphere around the particle.

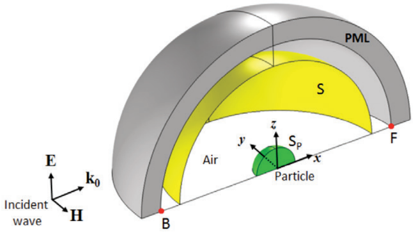

FIGURE 2. Model geometry for Mie scattering by a spherical particle.

COMPUTATIONAL ELECTROMAGNETICS

A computational model of Mie scattering was developed using COMSOL Multiphysics® and its RF Module. It solves for the scattering off of a dielectric, magnetic, or metal spherical particle with radius α. The model geometry is shown in Figure 2.

The air domain is truncated by a perfectly matched layer (PML) inserted to limit the extent of the model to a manageable region of interest. The solution inside the domain is not affected by the presence of the PML, which lets the solution behave as if the domain was of infinite extent. This layer absorbs all outgoing wave energy without any impedance mismatch that could cause spurious reflections at the boundary. The PML is useful in maintaining the solution at the desired level of accuracy and optimizing usage of computational resources. COMSOL also supports far-field calculations, which are done on the inner boundary of the PML domain where the near field is integrated. The surface *S* is used to calculate total scattered energy. An incident plane wave travels in the positive x-direction (see Figure 2), with the electric field polarized along the z-axis. Perfect magnetic conductor (PMC) and perfect electric conductor (PEC) boundary conditions are used on the x-z and x-y symmetry planes, respectively. The plane wave incident on the sphere is defined by its amplitude, wave vector in the air, and circular frequency. COMSOL conveniently provides all the necessary functionality to calculate scattering integrals. Scattering characteristics for the three types of particles considered are shown in Figures 3, 4, and 5. The results of the computational analysis show good agreement with available experimental results[1].

Simulation of Mie scattering problems enables visualization of the effects of small particles on an incident electromagnetic wave (see Figure 6) to allow better understanding of the interactions. ■

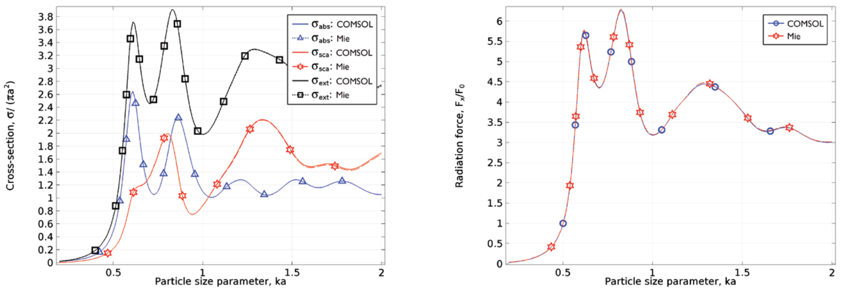

FIGURE 3. Cross-section parameters and radiation force for a dielectric particle with refractive index *n* = 5 – 0.4*j* and relative permeability μ = 1.

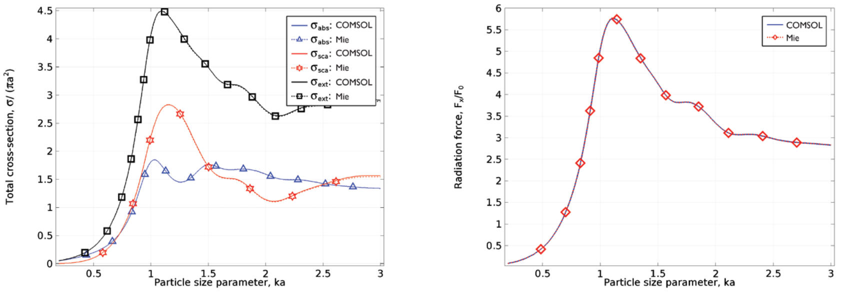

FIGURE 4. Cross-section parameters and radiation force for a magnetic particle with relative permittivity ε = 1 and relative permeability μ = 8 – 2*j*.

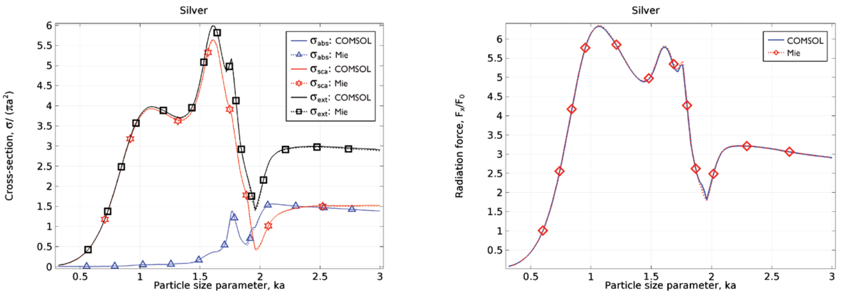

FIGURE 5. Cross-section parameters and radiation force for a silver particle with dielectric constants.

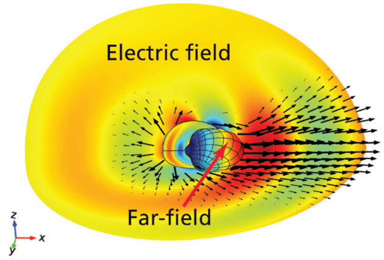

FIGURE 6. Distribution of the z-component of the electric field due to scattering of the incident electromagnetic wave by a particle of 0.1μm radius. The arrows show the time-averaged power flow of the relative fields at a frequency of 950 THz.

References

[1] Mätzler, C., *MATLAB Functions for Mie Scattering and Absorption, Version 2*, IAP Research Report, (Bern: InstitutfürangewandtePhysik, Universität, 2001), No. 2002-11.